Hoare-Style Verification for GP 2

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Graph Programming Language GP 2

- Experimental DSL for graphs and graph-like structures
- Rule-based, visual manipulation of graphs
- Non-deterministic
- Simple syntax and semantics to facilitate formal reasoning
Graph Programming Language GP 2

- Experimental DSL for graphs and graph-like structures
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- Simple syntax and semantics to facilitate formal reasoning
A graph is *transitive* if for every directed path \( v \leadsto v' \) with \( v \neq v' \), there is an edge \( v \rightarrow v' \).

**Main = link!**

\[
\text{link}(a, b, x, y, z : \text{list})
\]

where not edge(1, 3)
Program for transitive closure (cont’d)
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Program for transitive closure (cont’d)
Rule schemata ("attributed rules")

bridge(n: int; s, t: string; a: atom; x, y: list)

where \( n < 0 \) and not \( \text{edge}(1, 3) \)

- ':' is list concatenation
- LHS expressions are *simple*
- Variables in RHS and condition occur in LHS
- Host graph expressions are constants (integers, strings and lists)
- *Marked* nodes and edges: red, green, blue, shaded, dashed
Subtype hierarchy for labels

list
  ⊆ atom
  ⊇ int ⊇ string
    ⊆ char
Rule-schema application

\[
\begin{array}{c}
\text{Rule-schema application}\\
\end{array}
\]
Program for vertex colouring

Main = init!; inc!

init(x: atom)

\[
\begin{align*}
x & \quad \Rightarrow \quad x:1 \\
1 & \quad & 1
\end{align*}
\]

inc(x, y: atom; i: int; a: list)

\[
\begin{align*}
x:i & \quad \xrightarrow{a} \quad y:i \\
1 & \quad & 2
\end{align*} \quad \Rightarrow \quad 
\begin{align*}
x:i & \quad \xrightarrow{a} \quad y:i+1 \\
1 & \quad & 2
\end{align*}
\]
Program for vertex colouring

Main = init!; inc!

init(x: atom)

\[
\begin{array}{c c}
\text{x} & \Rightarrow & \text{x:1} \\
1 & & 1 \\
\end{array}
\]

inc(x, y: atom; i: int; a: list)

\[
\begin{array}{c c}
\text{x:i} & \rightarrow & \text{y:i} & \Rightarrow & \text{x:i} & \rightarrow & \text{y:i+1} \\
1 & a & 2 & & 1 & a & 2 \\
\end{array}
\]

Assumption: node labels in host graph are atoms
Program for vertex colouring (cont’d)
Program ::= Decl \{Decl\}

MainDecl ::= Main '=' ComSeq

ComSeq ::= Com \{'','\ Com\}

Com ::= RuleId
    | '{' [RuleId \{'','\ RuleId\}] '\}'
    | if ComSeq then ComSeq [else ComSeq]
    | try ComSeq [then ComSeq [else ComSeq]]
    | ComSeq '!'
Transition relation defined by SOS

Main = \{r_1, r_2\}; \{r_1, r_2\}; r! \\

r_1: 1 \Rightarrow 1 \quad r_2: 1 \Rightarrow 2

\langle \text{Main}, 1 \rangle \rightarrow \langle P, 2 \rangle \rightarrow \text{fail} \\
\downarrow \\
\langle P, 1 \rangle \rightarrow \langle r_1!, 1 \rangle \rightarrow \langle r_1!, 1 \rangle \rightarrow \ldots \\
\downarrow \\
\langle r_1!, 2 \rangle \\
\downarrow \\
2

where \( P = \{r_1, r_2\}; r! \)
Verification: Motivation

Motivated by applications of graph programs to defining semantics and analyses of languages and systems, e.g.

- Visual modelling/specification languages
- Model transformations
- Shape safety of pointer operations
- Concurrent asynchronous programming abstractions

Can existing “classical” verification tools help us?
Motivated by applications of graph programs to defining semantics and analyses of languages and systems, e.g.

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Can existing “classical” verification tools help us?

- Yes, but blow up in inherently symmetric & dynamic problems
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Can existing “classical” verification tools help us?

- Yes, but blow up in inherently symmetric & dynamic problems
- Alternatively: lift and tailor classical verification techniques to the domain of graphs and graph transformation
Verification: Our Approach

In particular: we are developing a theory to underpin assertional graph-based reasoning

- Graph/morphism-based assertion logic
- Hoare logics for programs
- Effective weakest precondition constructions for rules
- Exploits an algebraic characterisation of graph rewriting

\[ \vdash \{ \text{pre} \} \; P \; \{ \text{post} \} \]
Graph-Based Assertion Logic

We combine ideas from:

- Nested conditions (finite)
  - Express local graph structure via morphisms
- Classical FO logic; attributed graph constraints
  - Express properties of attributes
- MSO logic on graphs (in the presentation of Courcelle)
  - Express non-local properties (paths, cycles, ...)
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- MSO logic on graphs (in the presentation of Courcelle)
  - Express non-local properties (paths, cycles, ...)

Example: local graph structure and attributes

\[ \forall_L x, y. \forall \bigcirc_v \bigcirc_w [ x \neq y \implies \exists \bigcirc_v x \rightarrow [y]_w ] \]

"for all pairs of nodes, if their labels are distinct, then they are adjacent"
Example: non-local property

\[ \exists v \{ X, Y \} . \quad \forall \bullet _v \quad [ ( v \in X \lor v \in Y ) \land \neg ( v \in X \land v \in Y ) ] \]

\land \forall \bullet _v \bullet _w \quad [ \exists \bullet _v \to \bullet _w \implies \neg ( v \in X \land w \in X ) \land \neg ( v \in Y \land w \in Y ) ]

"the graph is 2-colourable (bipartite)"
Graph-Based Assertion Logic

Example: **non-local** property

\[ \exists_v \forall X, Y. \ (v \in X \lor v \in Y) \land \neg (v \in X \land v \in Y) \land \forall v \exists_v \exists_w [ \exists_v \exists_w \Rightarrow \neg (v \in X \land w \in X) \land \neg (v \in Y \land w \in Y) ] \]

"the graph is 2-colourable (bipartite)"

Example: **attributes** and **non-local** property

\[ \exists \exists_v [ \neg \exists_v \exists_v \exists_w ] \land \forall x. \ (x > 0 \Rightarrow \exists \exists_v \exists_w [ path(v,w) ] ] \]

"there is an (arbitrary-length) path from every positive integer-labelled node to a unique red node"
Hoare-Style Proof Calculi

Proof rules for (partial/total) correctness of GP 2 constructs, e.g.

\[
\begin{array}{c}
\text{[comp]} & \{c\} \quad P \quad \{e\} \quad Q \quad \{d\} & \quad \text{[!]} & \quad \{\text{inv}\} \quad R \quad \{\text{inv}\} \\
& \{c\} \quad P; \quad Q \quad \{d\} & & \{\text{inv}\} \quad R! \quad \{\text{inv} \land \neg \text{App}(R)\}
\end{array}
\]

Core of the calculi:

\[
\vdash \{\text{Pre}(r, post)\} \quad r \quad \{post\}
\]
Hoare-Style Proof Calculi

Proof rules for (partial/total) correctness of GP 2 constructs, e.g.

\[
\begin{align*}
\text{[comp]} & \quad \frac{\{c\} P \{e\} \quad \{e\} Q \{d\}}{\{c\} P ; Q \{d\}} \\
\text{[!]} & \quad \frac{\{\text{inv}\} R \{\text{inv}\}}{\{\text{inv}\} R! \{\text{inv} \land \neg \text{App}(R)\}}
\end{align*}
\]

Core of the calculi:

\[ \vdash \{ \text{Pre}(r, \text{post}) \} \ r \ \{ \text{post} \} \]

- Extended from Habel/Pennemann’s construction to handle attributes, MSO logic, and path predicates
Example: Colouring

Main = init!; inc!

init(x: atom)

\[
\begin{array}{c}
\text{x} \\
1
\end{array} \Rightarrow \begin{array}{c}
\text{x:1} \\
1
\end{array}
\]

inc(x, y: atom; i: int; a: list)

\[
\begin{array}{c}
n x:i \\
1
\end{array} \xrightarrow{a} \begin{array}{c}
y:i \\
2
\end{array} \Rightarrow \begin{array}{c}
x:i \\
1
\end{array} \xrightarrow{a} \begin{array}{c}
y:i+1 \\
2
\end{array}
\]

\[c = \forall_L a. \forall v [\text{atom}(a)]\]

\[d \land \lnot \text{App} \{\text{inc}\} = \forall_L a. \forall v [\exists_A b. \exists_I c. a = b : c \land c \geq 1] \land \lnot \exists_L k. \exists_A x, y. \exists_I i. \exists_v \begin{array}{c}
x:i \\
v
\end{array} \xrightarrow{k} \begin{array}{c}
y:i \\
w
\end{array}\]
Example: Colouring

\[
\begin{align*}
\text{[ruleapp]} & \quad \frac{\text{Pre}(\text{init}, e)}{\text{init} \{ e \}} \\
\text{[cons]} & \quad \frac{\text{init} \{ e \}}{\text{init} \{ e \}} \\
\text{[!] } & \quad \frac{\text{init} \{ e \} \text{ init} \{ e \wedge \neg \text{App}\{\text{init}\} \}}{\text{init} \{ e \} \text{ init} \{ e \} \\
\text{[cons]} & \quad \frac{\text{init} \{ d \}}{\text{init} \{ d \} \\
\text{[comp]} & \quad \frac{\text{init} \{ d \}}{\text{init} \{ d \} \\
\quad \quad \vdash \text{par} \{ c \} \text{ init}; \text{ inc} \{ d \wedge \neg \text{App} \{ \text{inc} \} \}
\end{align*}
\]

\[
c = \forall_L a. \forall \bigotimes_v [\text{atom}(a)]
\]

\[
d \land \neg \text{App} \{ \text{inc} \} = \forall_L a. \forall \bigotimes_v [\exists_A b. \exists_I c. a = b : c \land c \geq 1] \\
\land \neg \exists_L k. \exists_A x, y. \exists_I i. \exists_{x,i} (k \xrightarrow{v} y:i) \\
\land \neg \exists_L k. \exists_A x, y. \exists_I i. \exists_{x,i} (w \xrightarrow{v} y:i)
\]
Example: Colouring

$$c = \forall_L a. \forall_v \bigotimes_v \left[ \text{atom}(a) \right]$$

$$d \land \neg \text{App}(\{\text{inc}\}) = \forall_L a. \forall_v \bigotimes_v \left[ \exists_A b. \exists_I c. a = b : c \land c \geq 1 \right] \land \neg \exists_L k. \exists_A x, y. \exists_I i. \exists x : i_v k \rightarrow y : i_w$$

$$\text{Pre}(\text{inc}, d) = \forall_L k. \forall_A x, y. \forall_I i. \forall x : i_v k \rightarrow y : i_w$$

$$[i \geq 1 \land \forall_L a. \forall x : i_v k \rightarrow y : i_w]$$

$$[\exists_A b. \exists_I c. a = b : c \land c \geq 1]$$
Open problems

Our calculi are sound with respect to the GP 2 semantics

But: assertion logic is not expressive

- Are there expressive extensions?
- Relative completeness (despite not expressive)?
- Proof rules for programs with composite tests / loop bodies?
Future Work

- Implementation for user-guided proofs
- Incorporating tracking information
- Automatic confluence and termination checking
- Applying to GROOVE’s control programs; CEGAR