Concepts of Concurrent Computation
Spring 2015
Lecture 8: Correctness Conditions

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Today's agenda

1. concurrent objects and correctness

2. quiescent and sequential consistency

3. linearizability

Terminology: concurrent objects

- a concurrent object is a data object shared by concurrent processes

  => *has a type defining possible values, and primitive methods that provide the only means of creation/manipulation*
  => *e.g. a shared data structure, a shared message queue, ...*

- a concurrent system is a collection of sequential processes that communicate through concurrent objects

Specifying correctness of operations

- in a sequential system, it is easy to specify the behaviour of methods

  => *pre- and postconditions*

$$\{pre\}\ q.op\ \{post\}$$

  => *methods cannot be called on objects that are in an "intermediate state"*

- in a concurrent system, need to accommodate interleavings of method invocations

What does it mean for concurrent objects to be correct?

- typically boils down to some notion of equivalence with sequential behaviour

- consider a simple, lock-based concurrent FIFO queue

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public void enq(int x) throws
FullException {
  lock.lock();
  try {
    if <<queue full>>
      { throw new FullException(); }
    <<enqueue x>>
    } finally {
      lock.unlock();
}}
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```
public int deq() throws
EmptyException {
  lock.lock();
  try {
    if <<queue empty>>
      { throw new EmptyException(); }
    <<dequeue x>>
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      lock.unlock();
}}
```
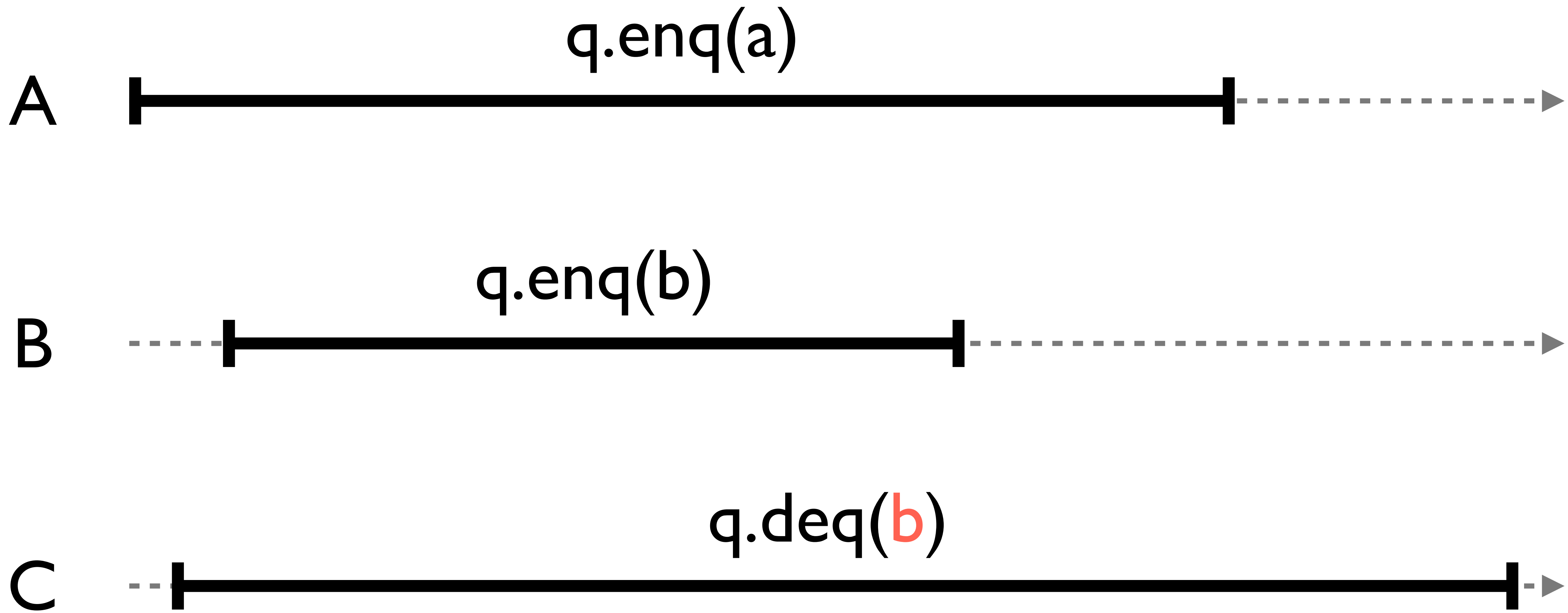

A lock-based concurrent FIFO queue



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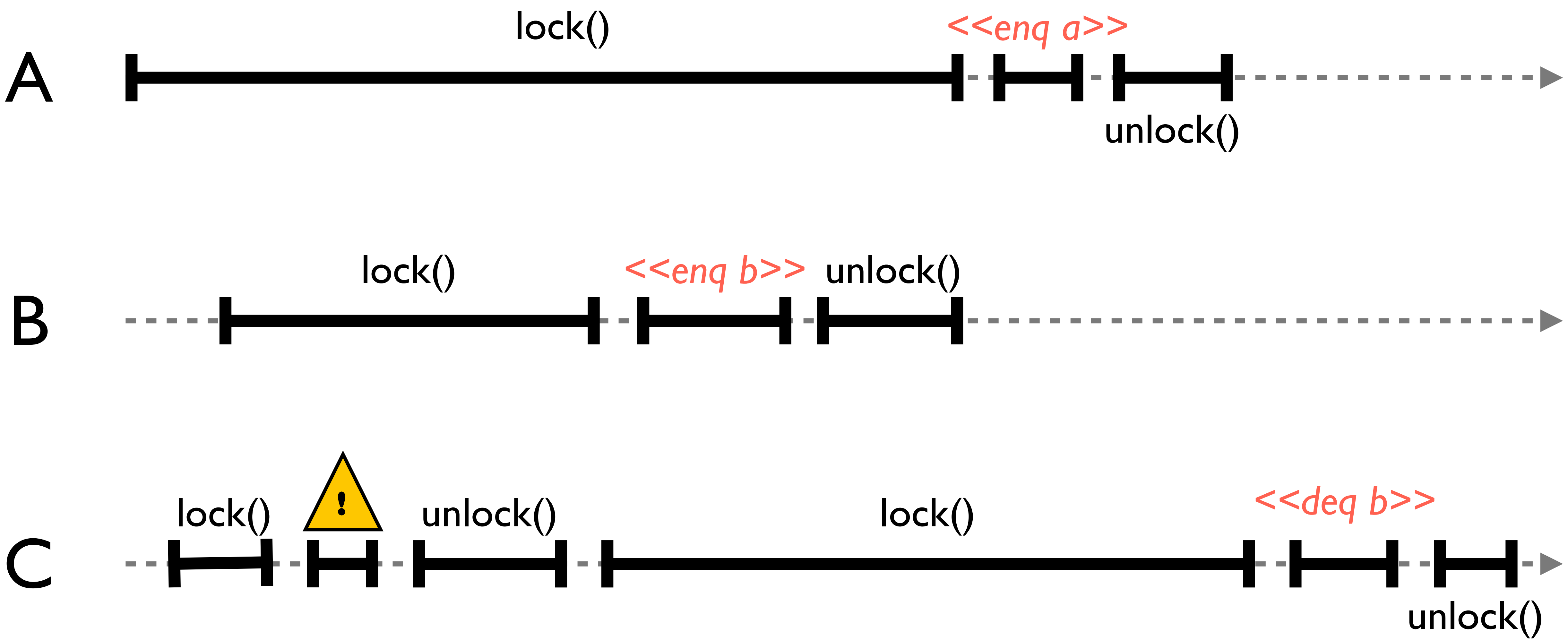

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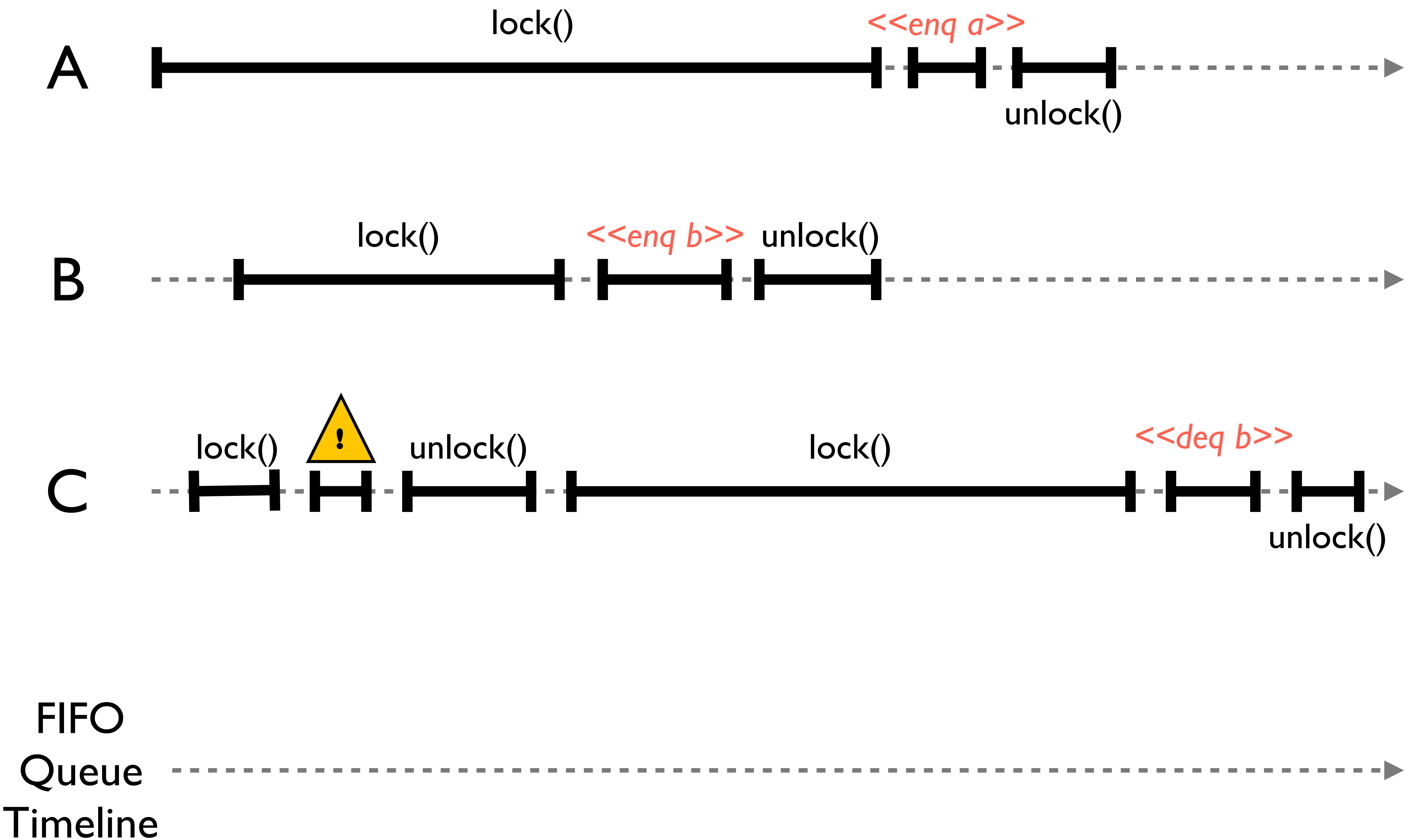

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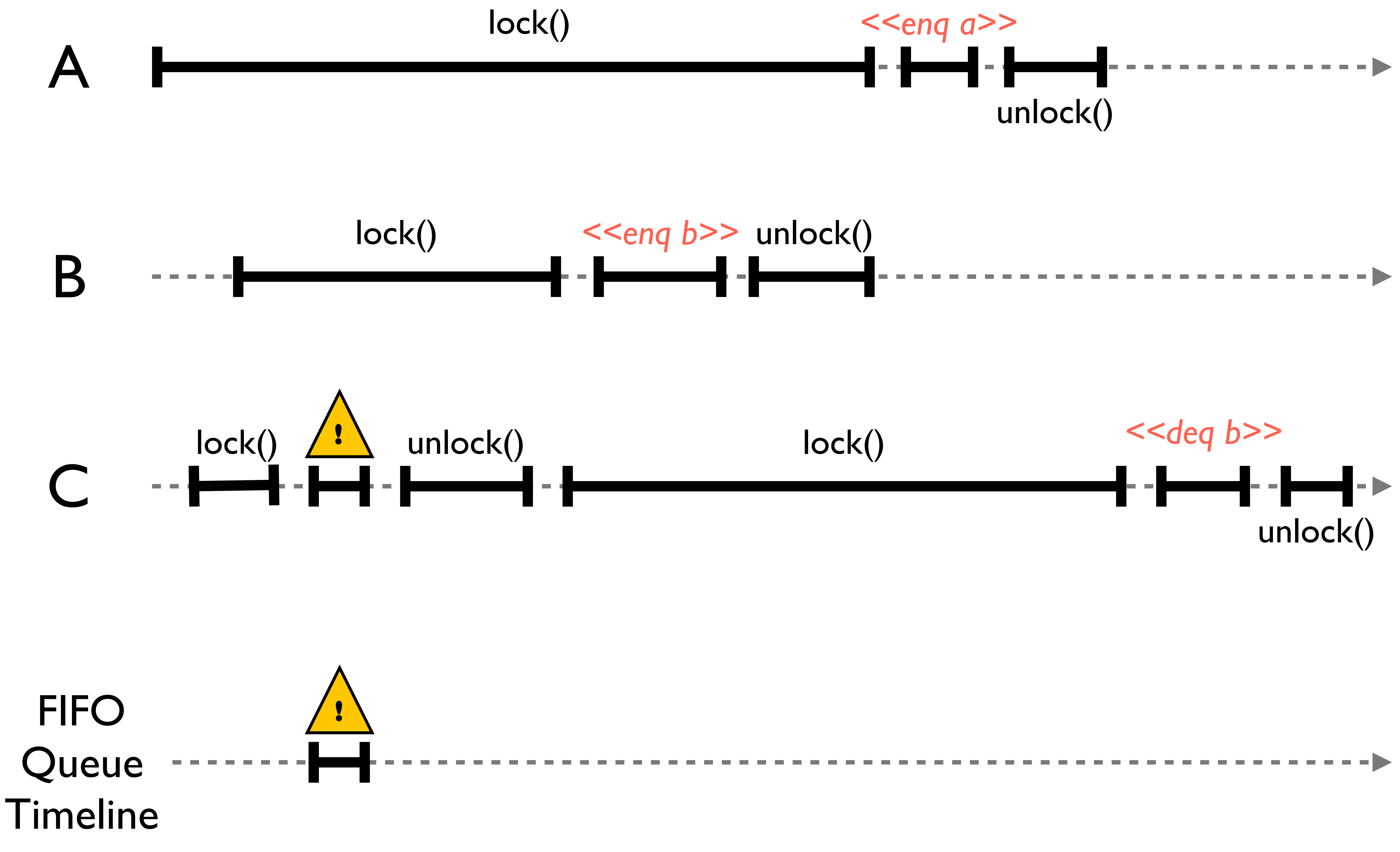

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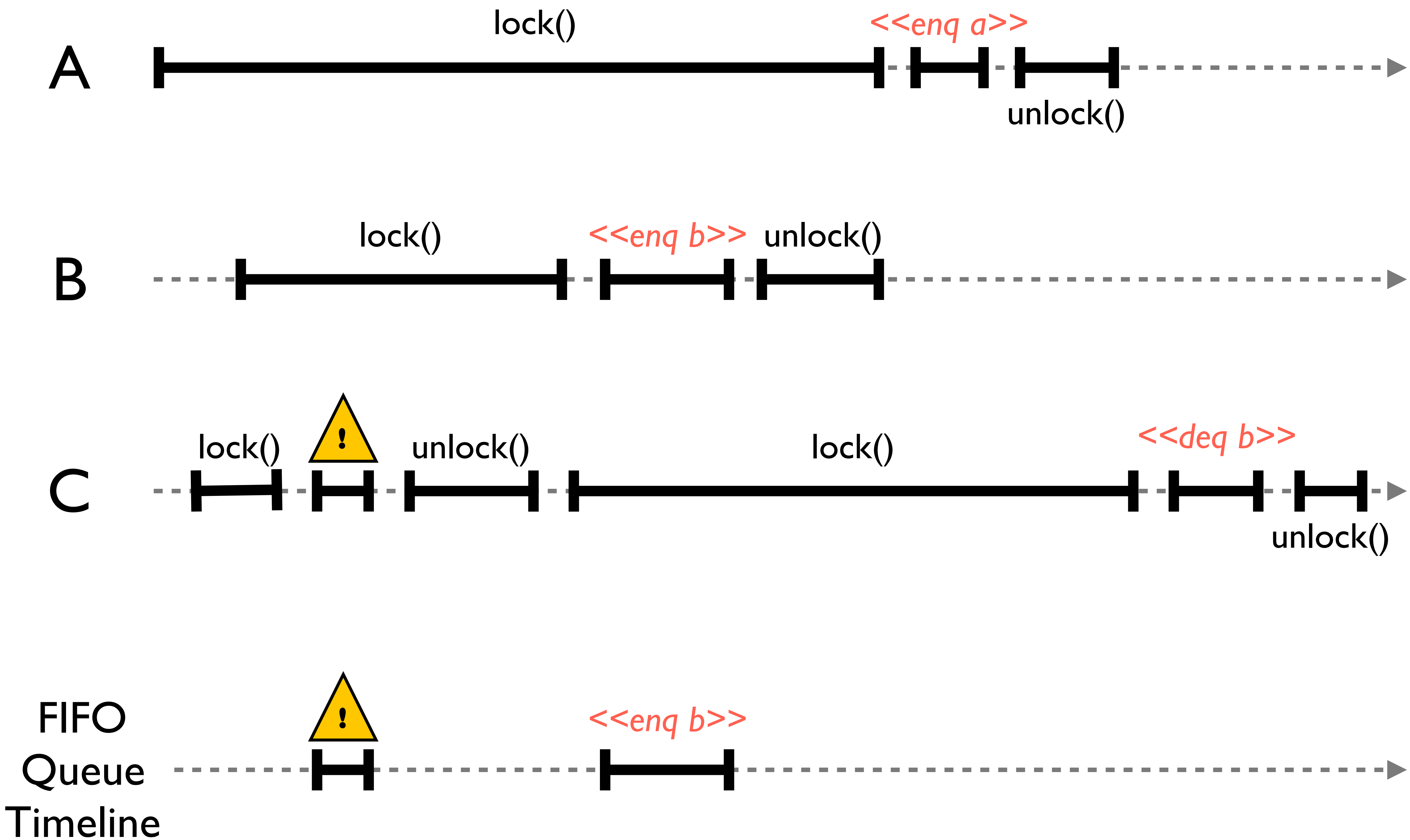

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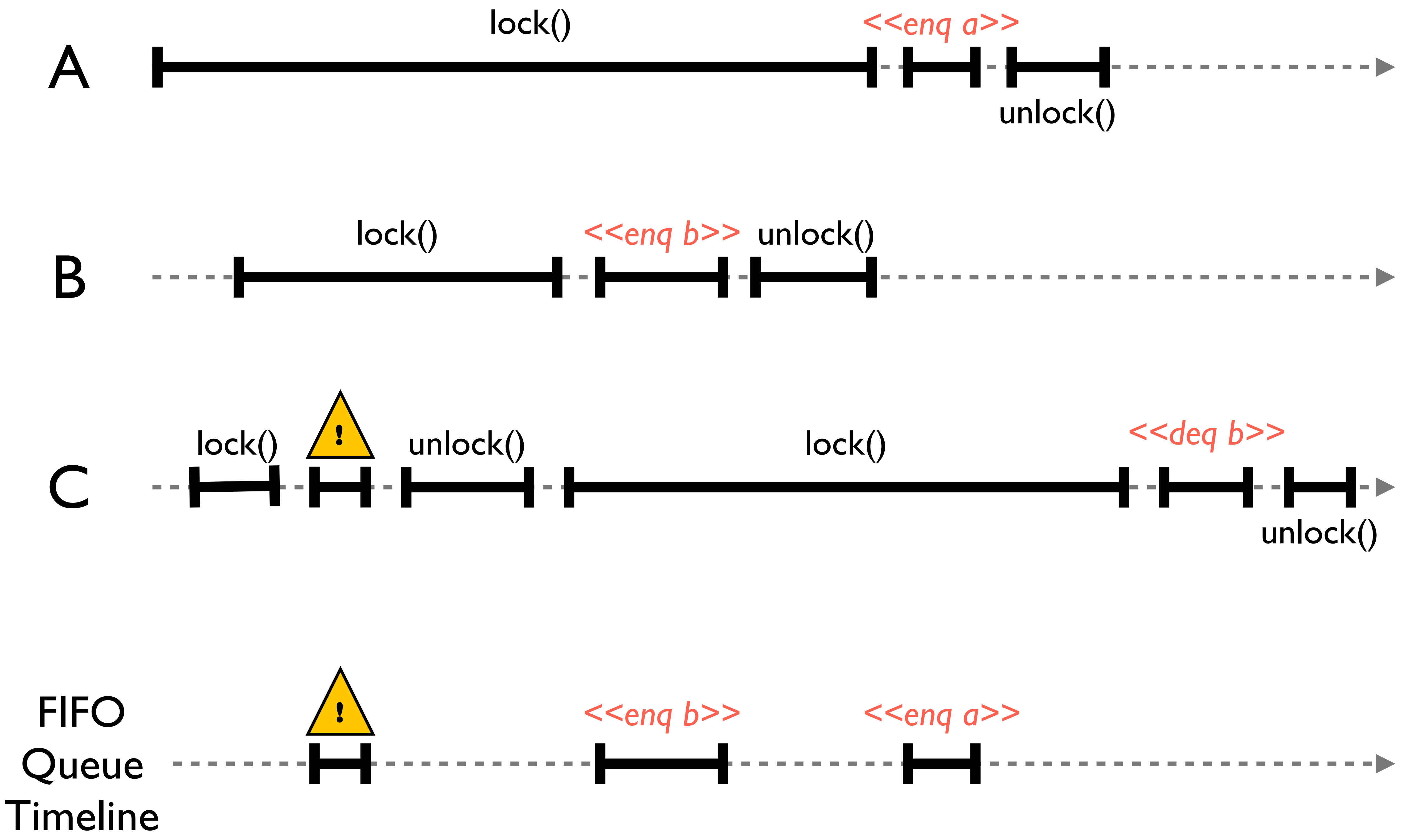

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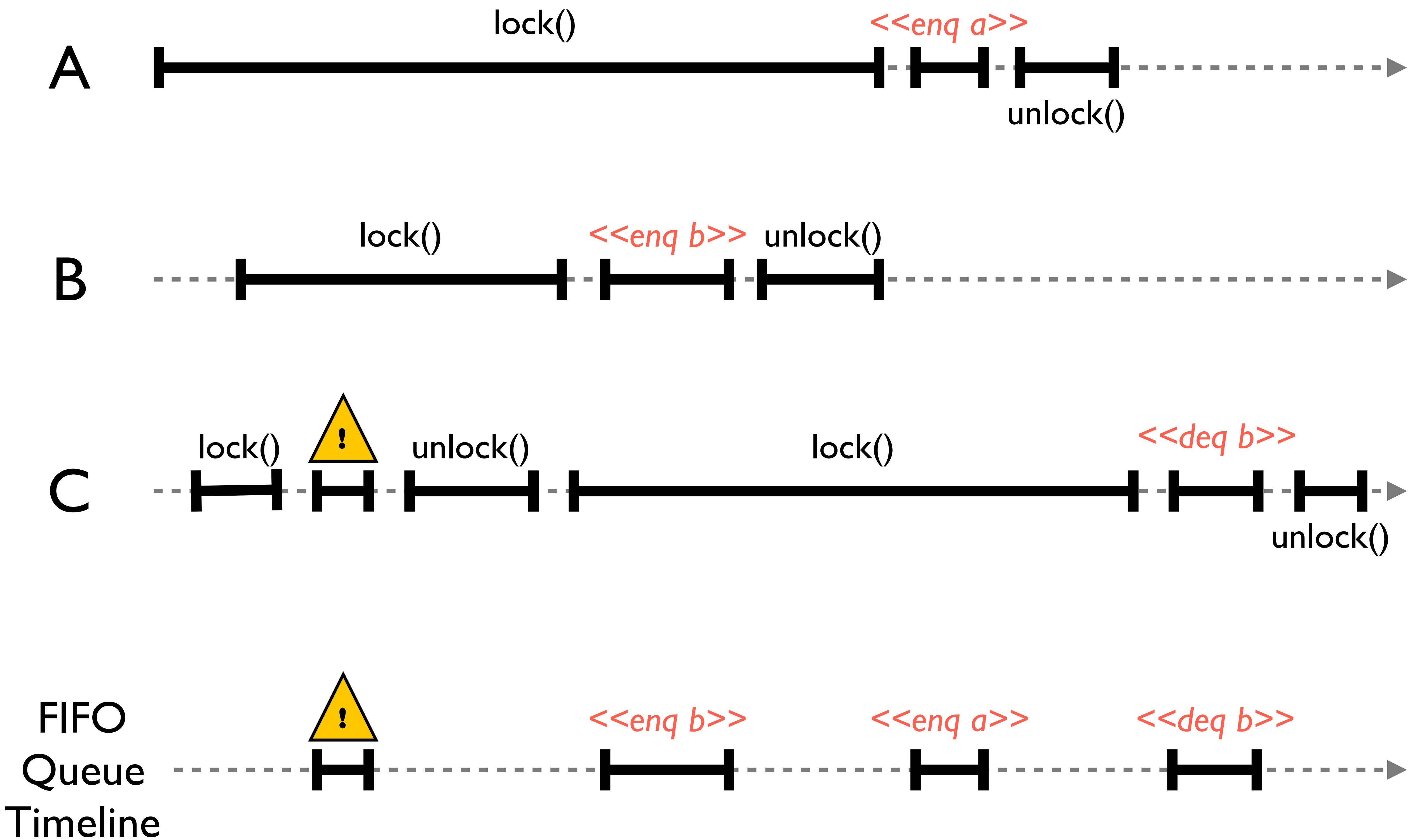

What happens if we drop the locks?

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are there circumstances in which this queue can be correct?

what does "correct" mean?

Reasoning about concurrent objects: a principle

- concurrent objects may have methods with finer-grained locking or no locking at all

- need to be able to specify and reason about implementations without relying on method-level locking

- but the example illustrates an important principle:

> *it's easier to reason about concurrent objects if we can map their concurrent executions to sequential ones*

Which "equivalences" with sequential behaviour do we care about?

- do we care about program order, fairness, ...?

- in practice, different applications require different "strengths" of correctness conditions

  => *print job queue for a lightly loaded printer*
  => *banking server (e.g. transfer money from savings; withdraw £50)*
  => *stock-trading server*

We will consider three correctness conditions

strength of condition

quiescent consistency

=> *whenever an object becomes quiescent, then the execution so far is equivalent to some sequential execution of the completed calls*

sequential consistency

=> *method calls should appear to take effect in a sequential order consistent with the program order*

linearizability

=> *each method call should appear to take effect instantaneously at some moment between its invocation and response*

Next on the agenda

1. concurrent objects and correctness ✓
2. quiescent and sequential consistency
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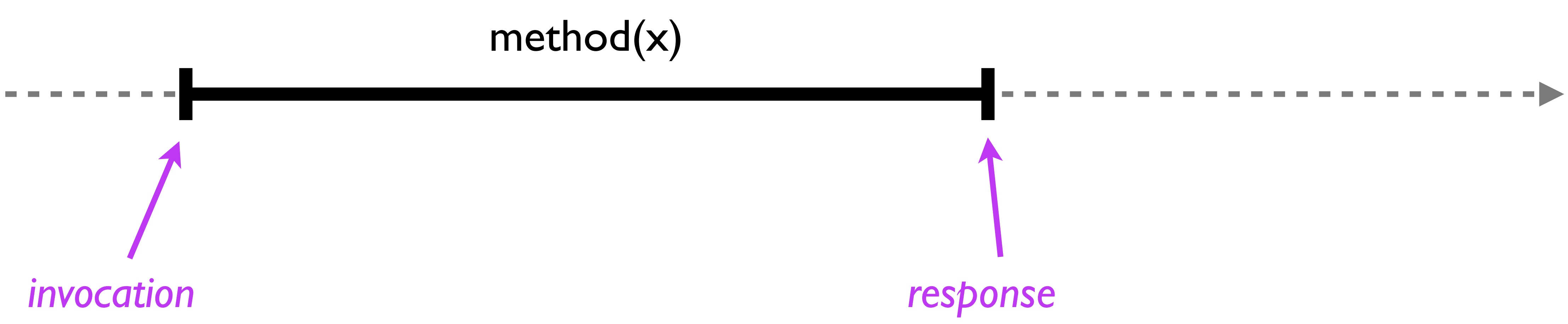
Terminology: method calls

- individual threads sequentially execute method calls that have invocation and response events

- a method is pending if its call has occurred, but not its response



What conditions / restrictions do we need?

- let's derive some principles from examples of unacceptable behaviours

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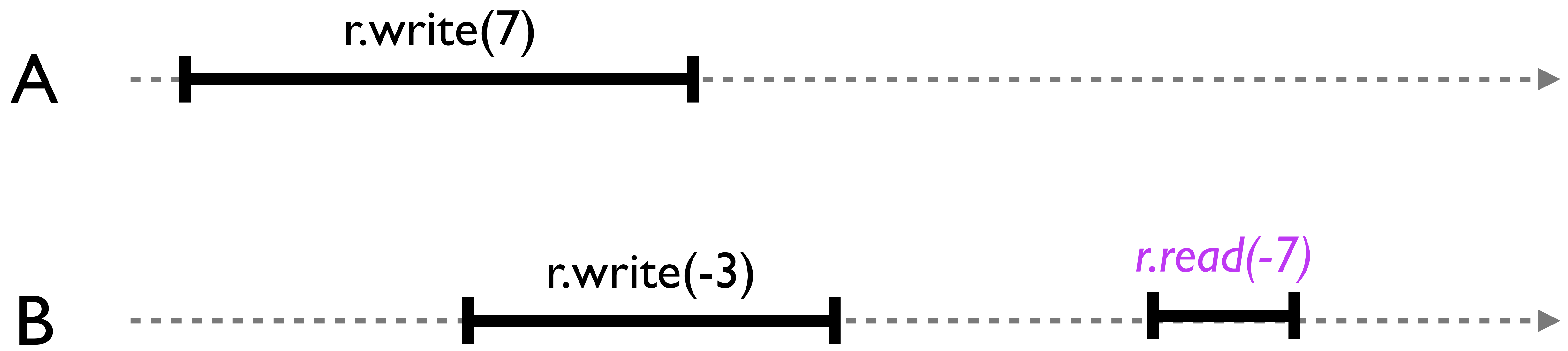

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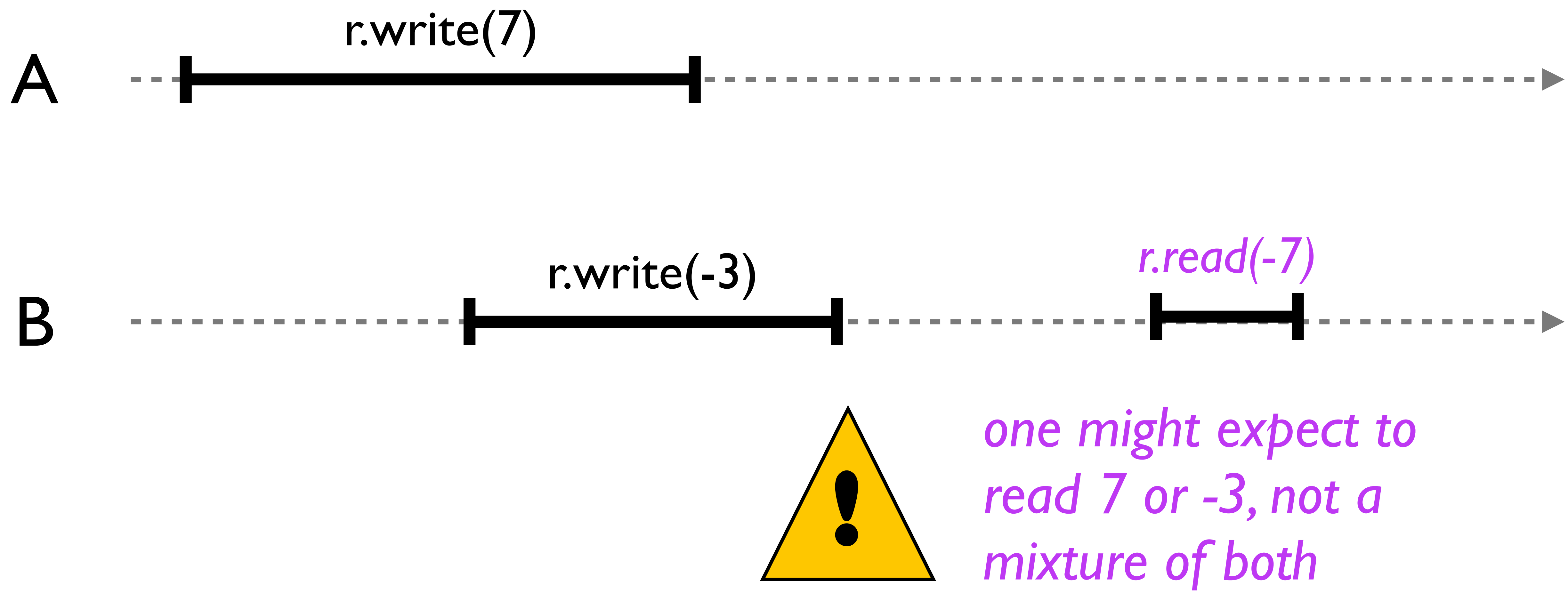

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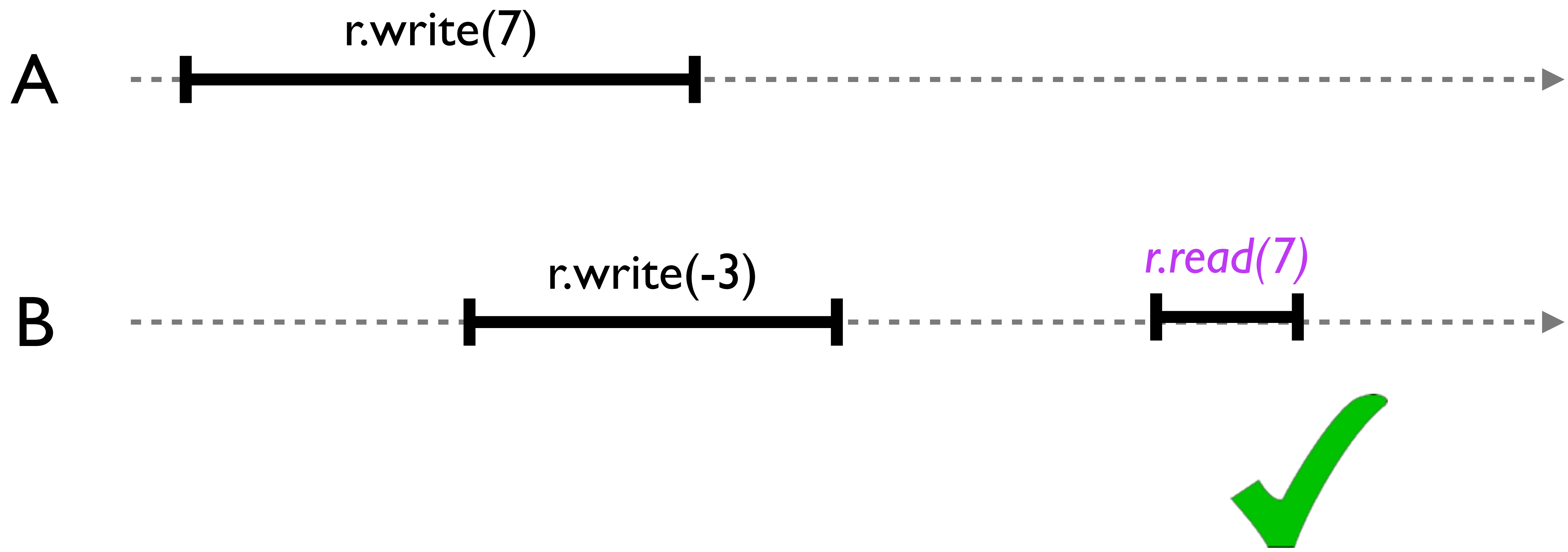

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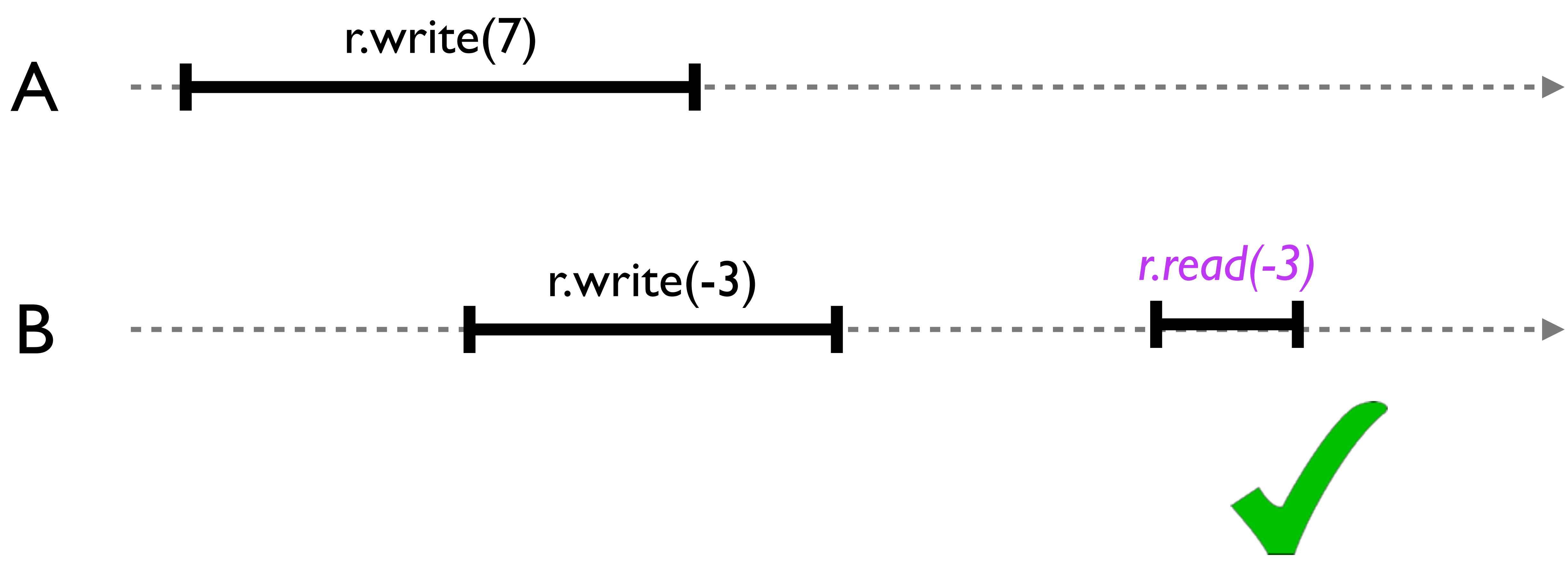

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too weak alone!

permits, e.g. readers to always return the object's initial state

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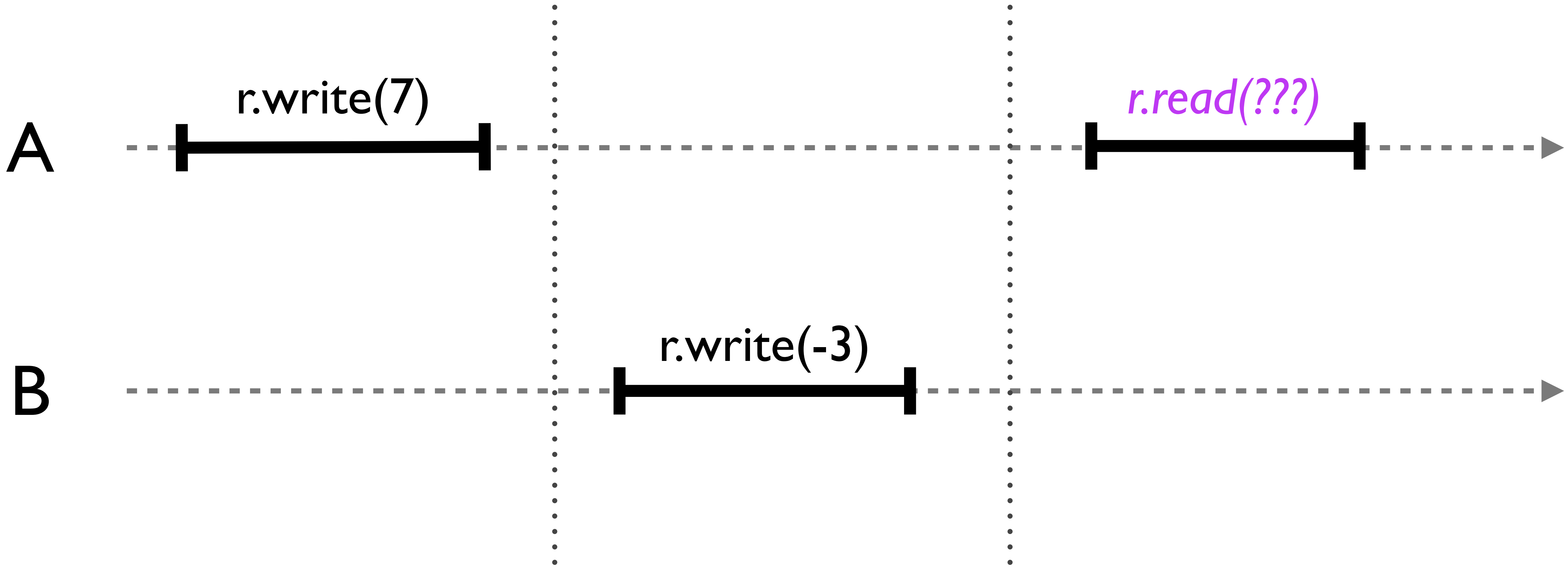

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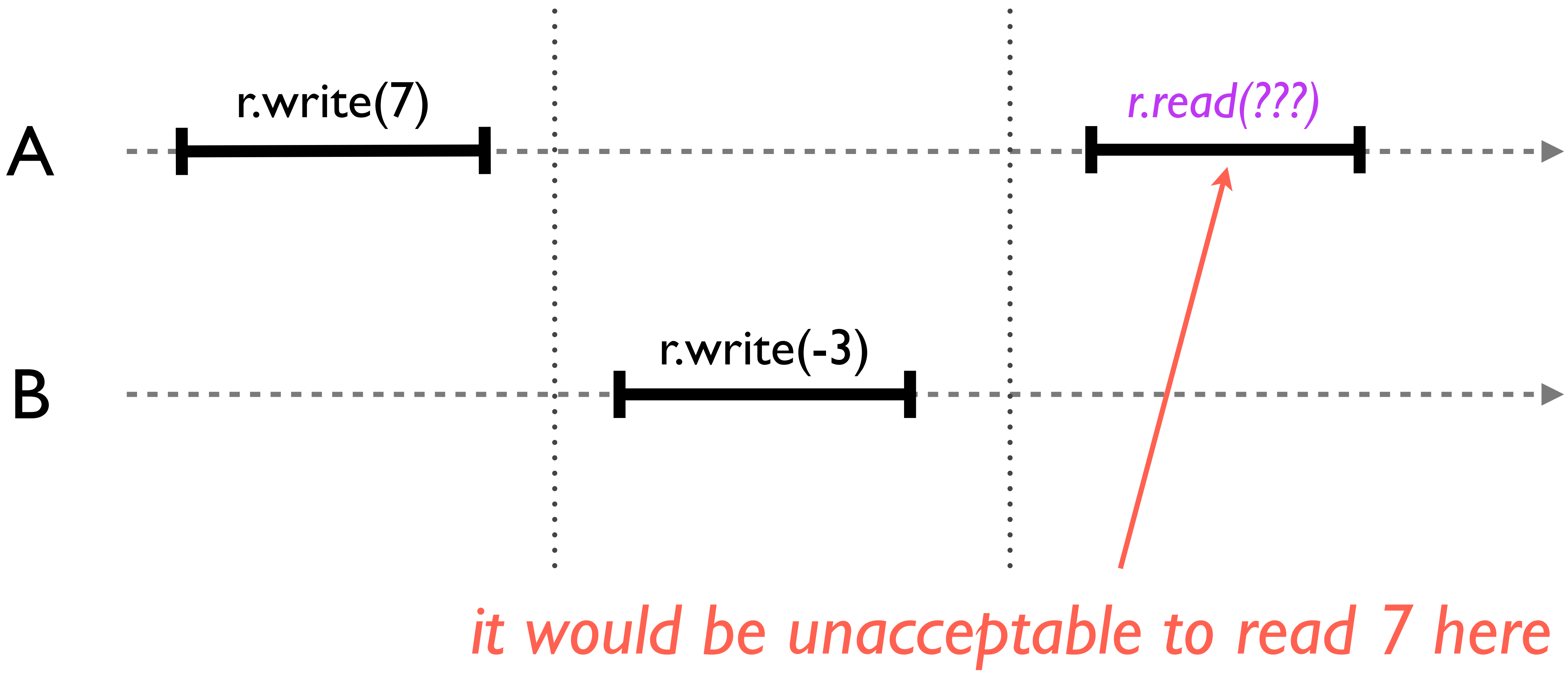


Quiescent consistency

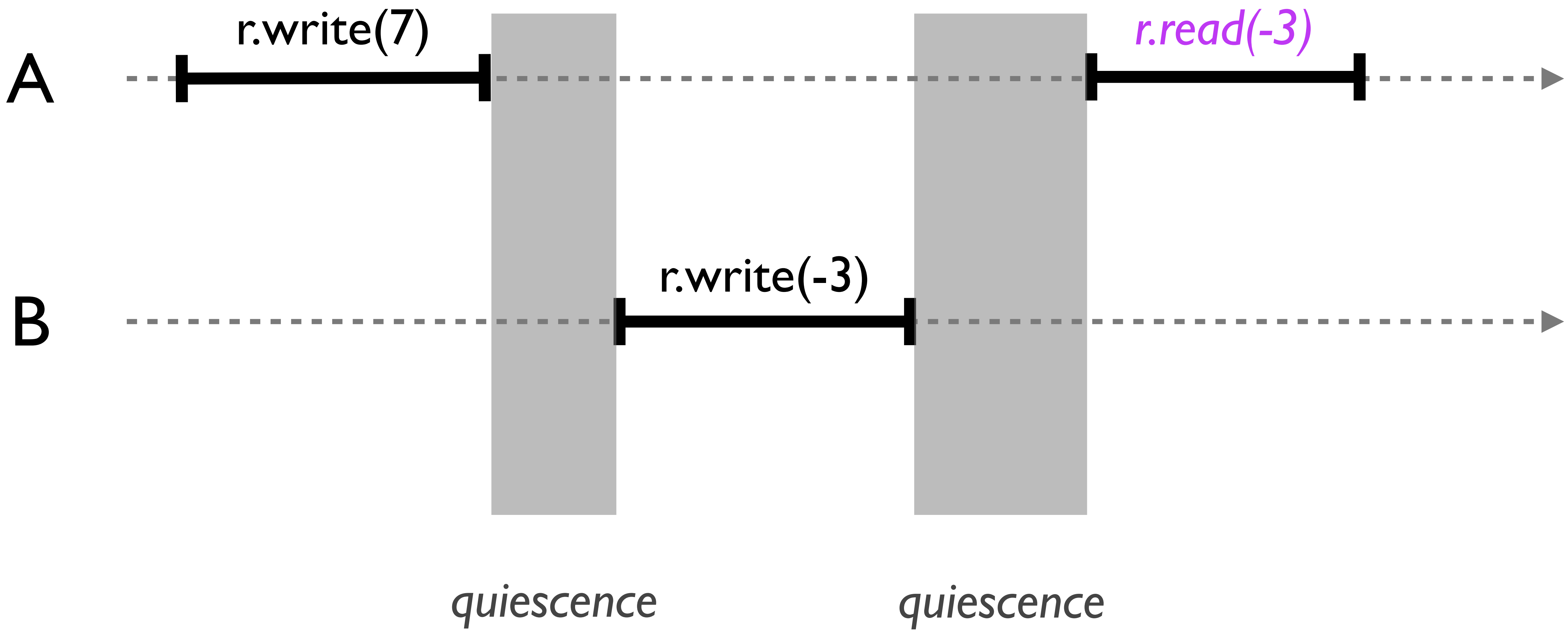
method calls should appear to happen in a one-at-a-time, sequential order

+

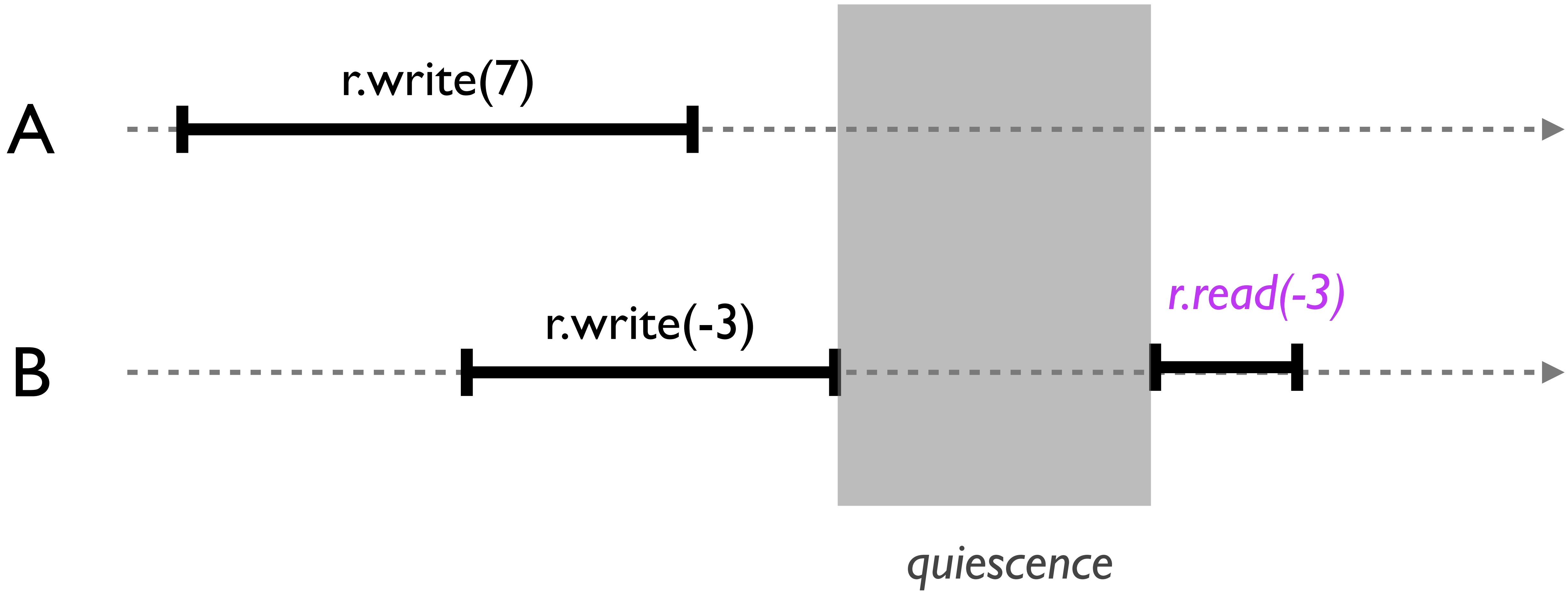
method calls separated by a period of quiescence should appear to take effect in their real-time order

NB: an object is quiescent if it has no pending method calls

Quiescent consistency



Quiescent consistency



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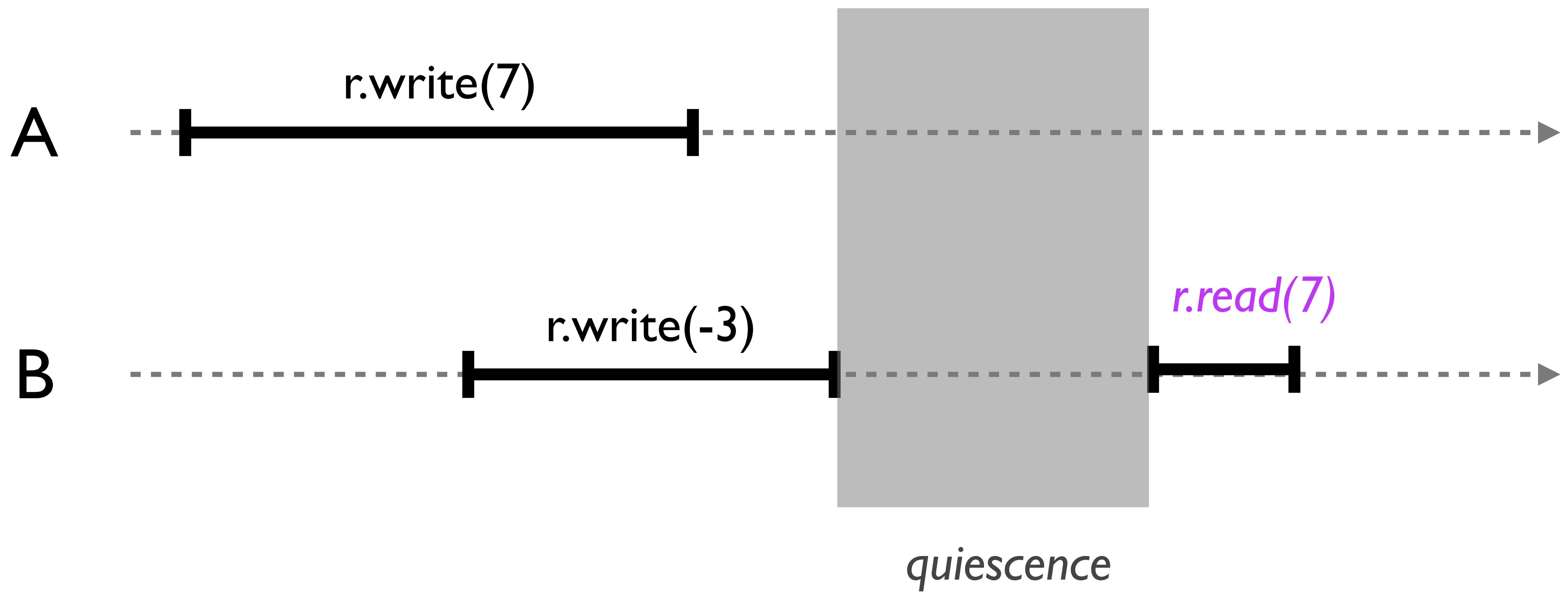

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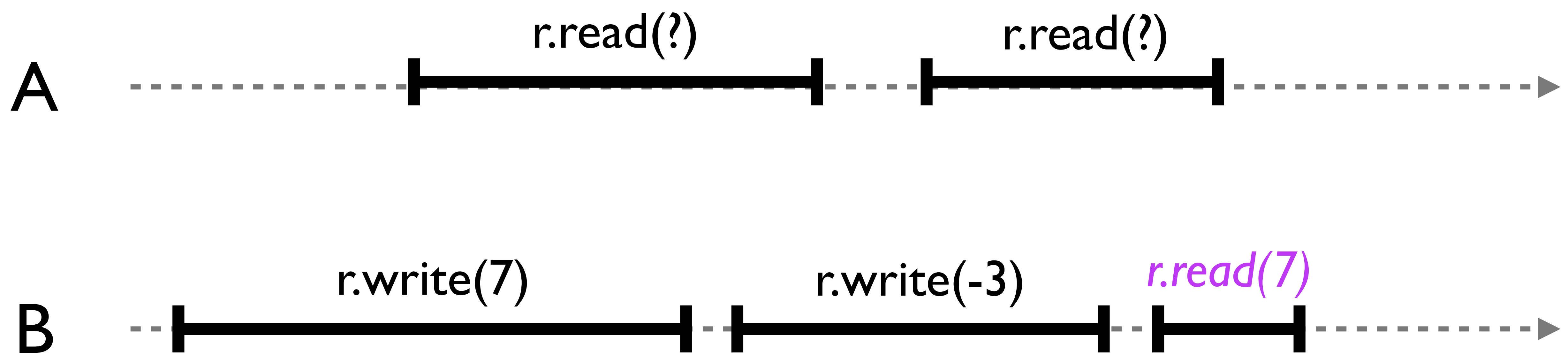

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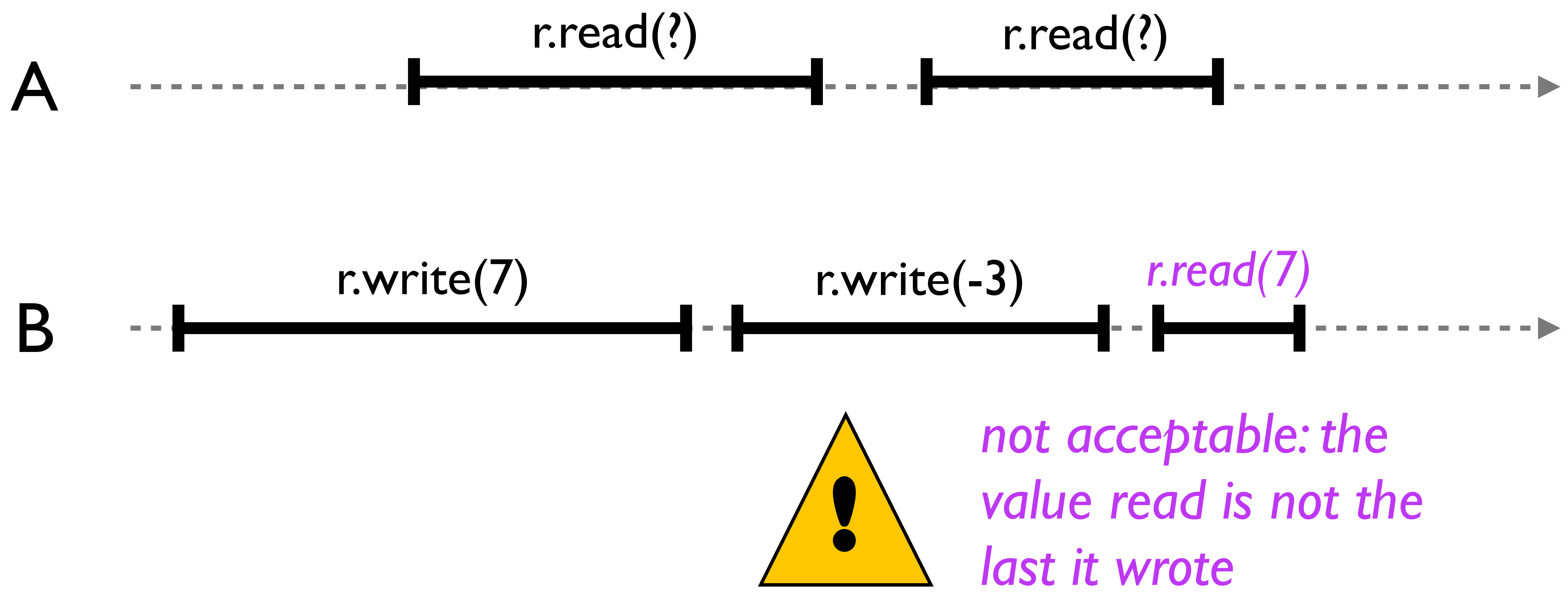

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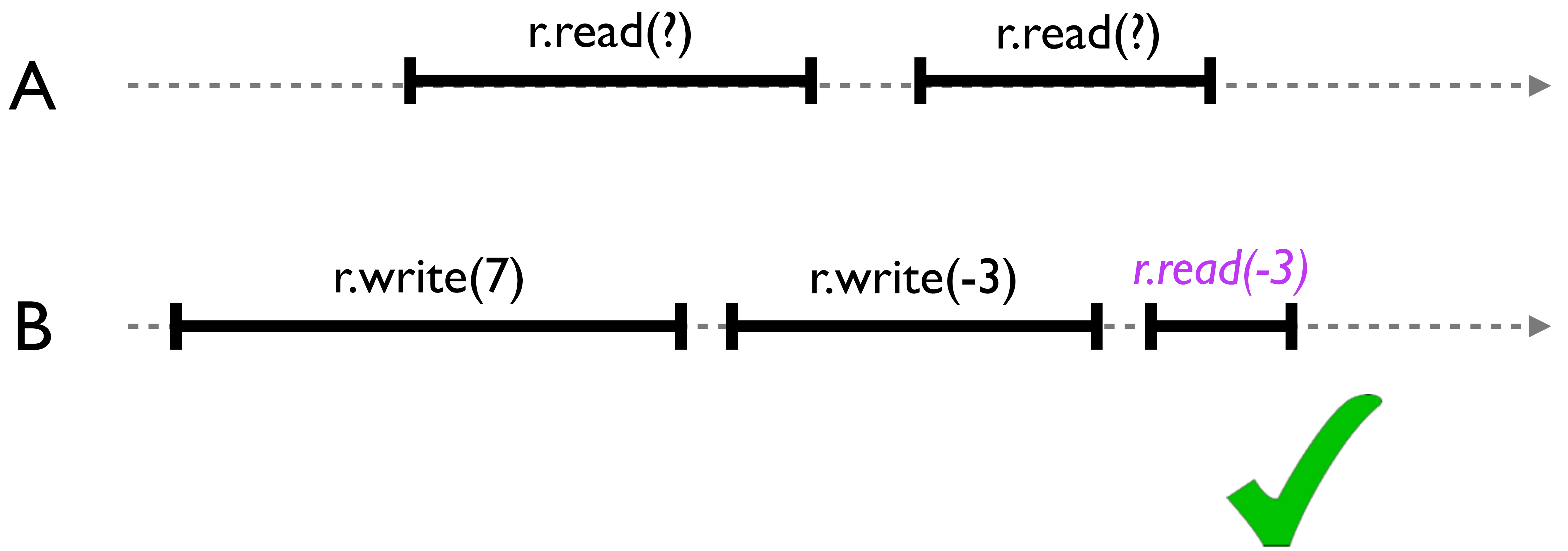
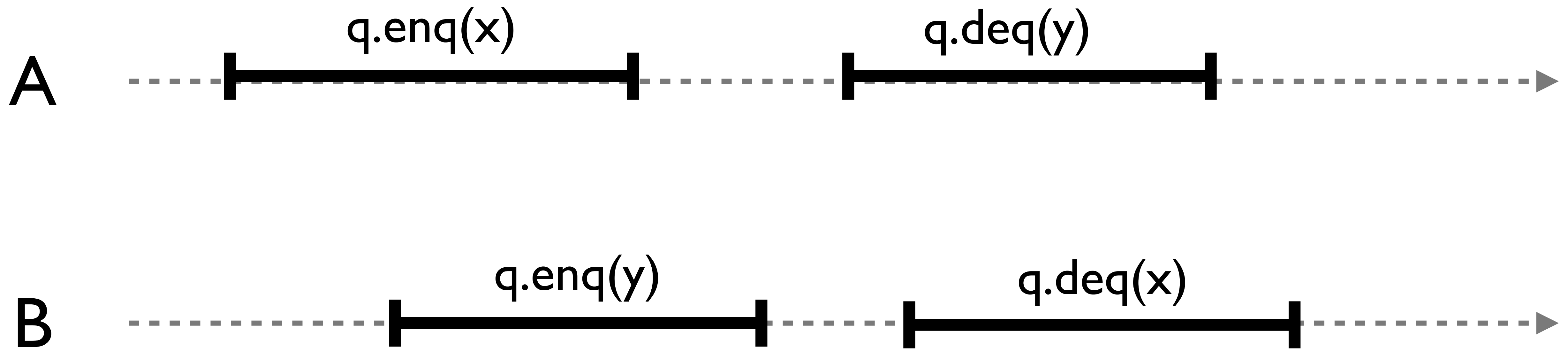
i.e. in any concurrent execution, there is a way to order the method calls sequentially so that they are (1) consistent with program order; and (2) meet the object's sequential specification

Sequential consistency



Is this execution sequentially consistent?



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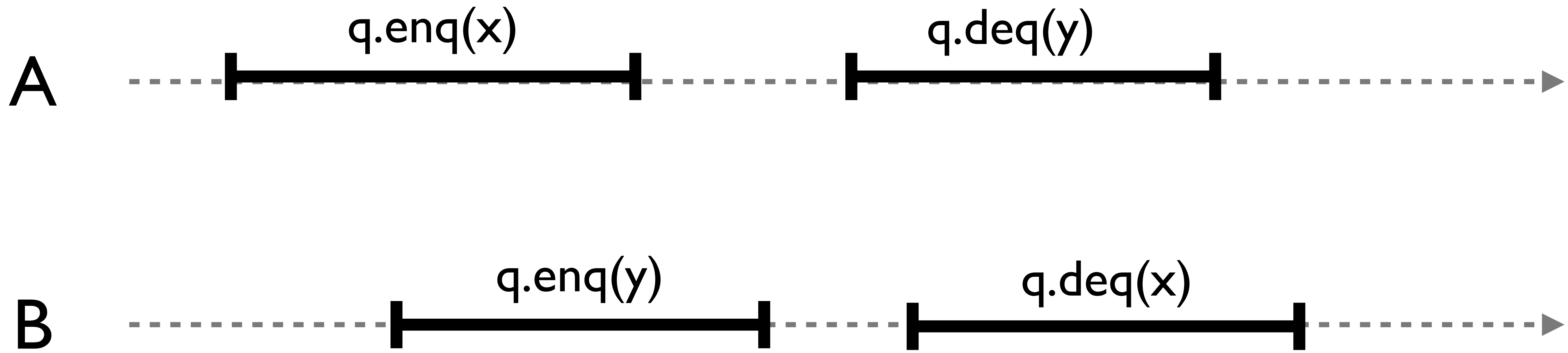

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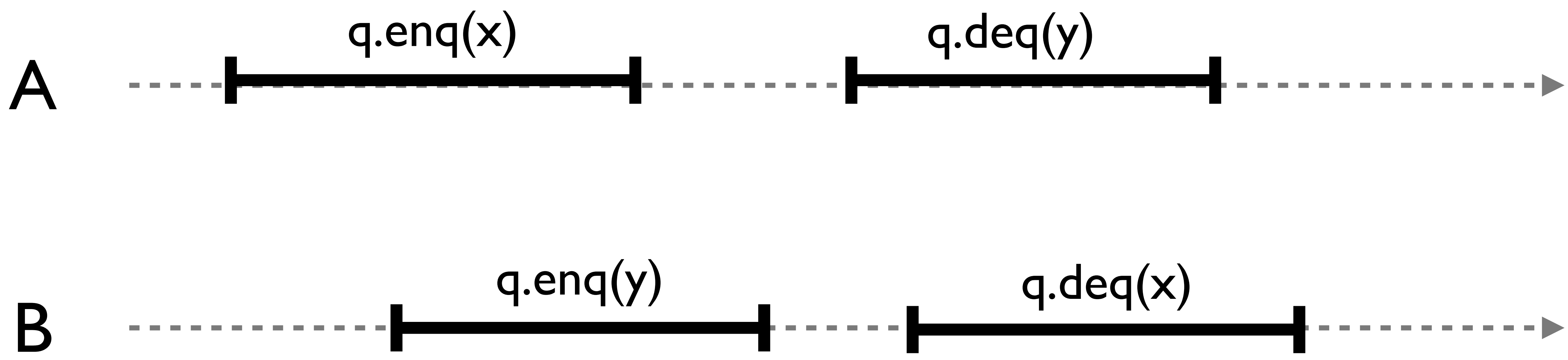

yes! two possible consistent orderings

A: enqueues x
B: enqueues y
B: dequeues x
A: dequeues y

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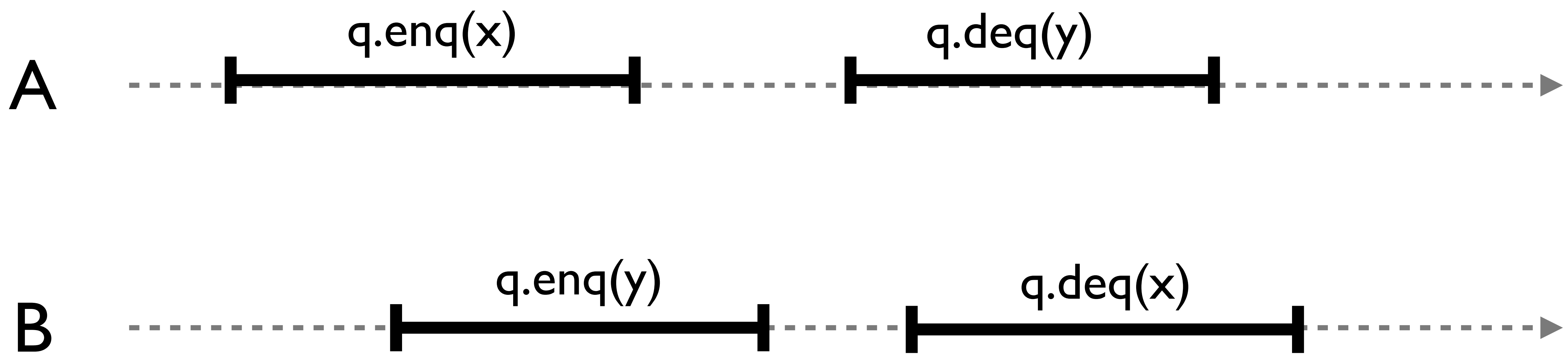

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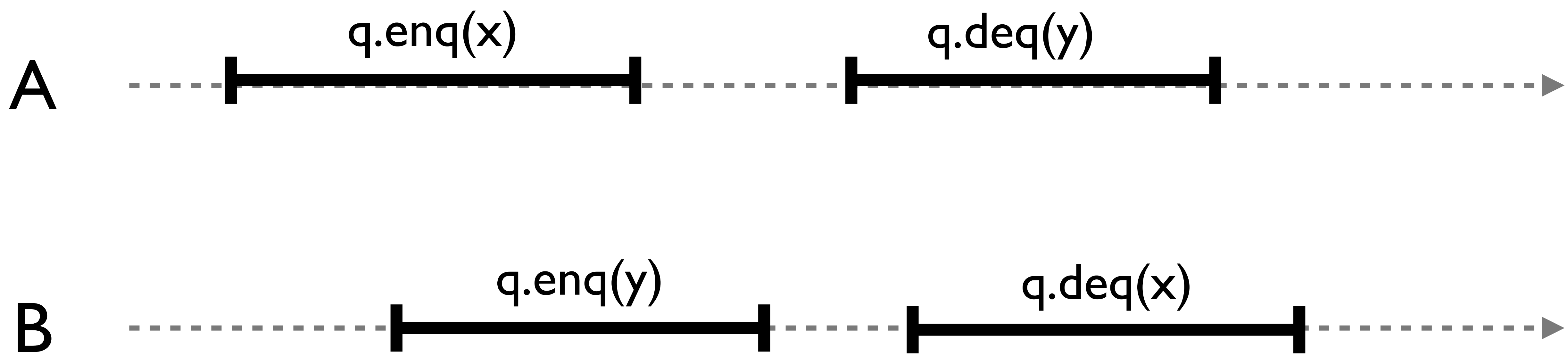

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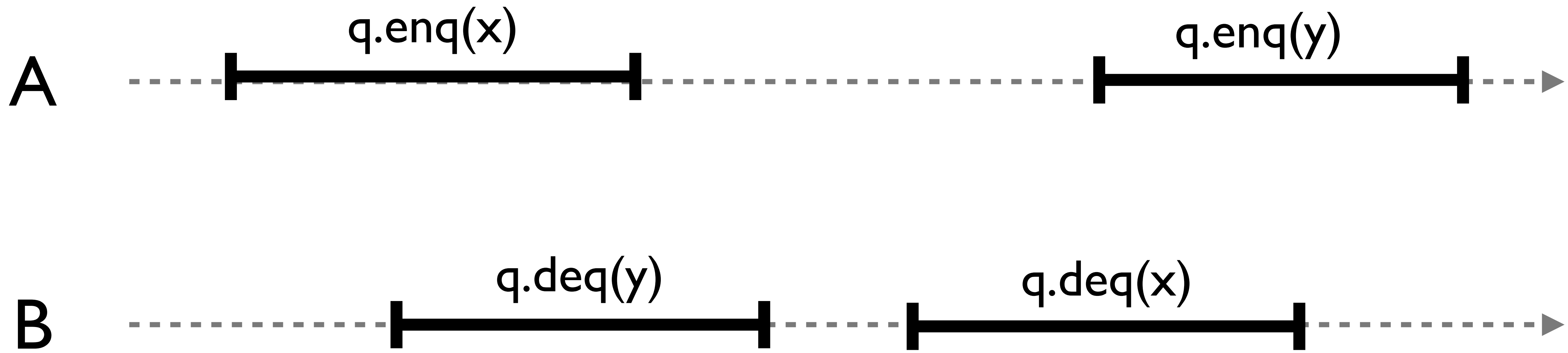
sequential program order preserved!

Is this execution sequentially consistent?

(from Huisman)



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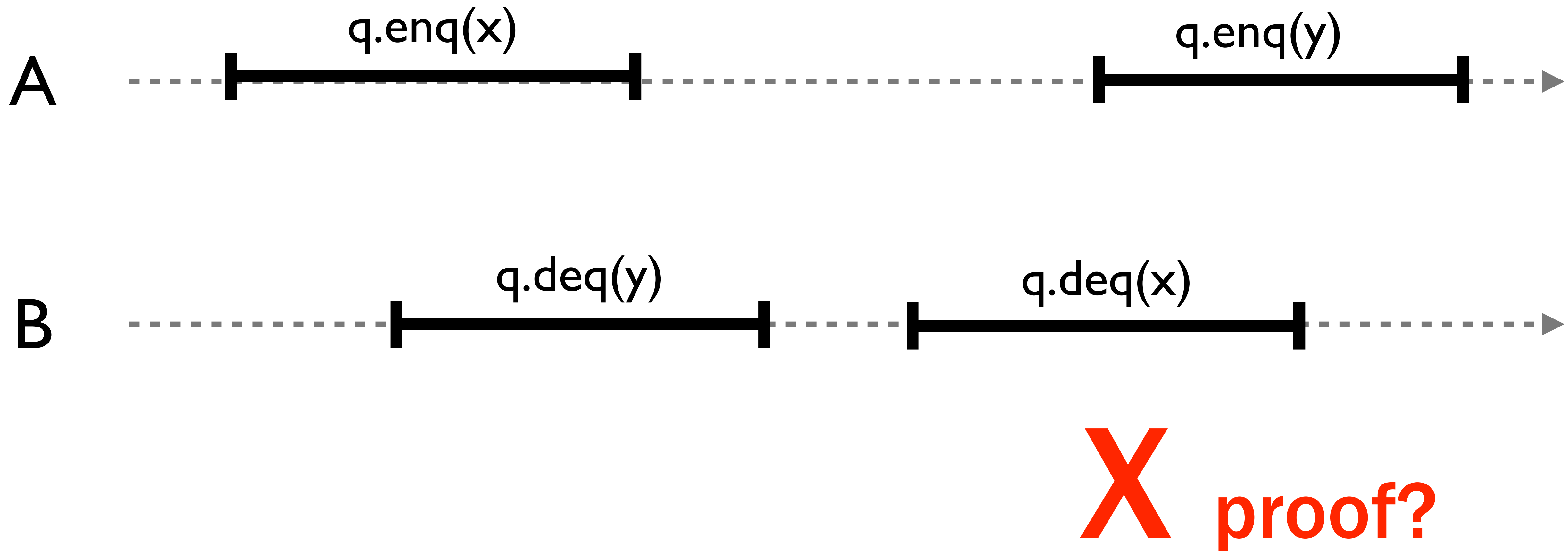


Quiescent vs. sequential consistency

- quiescent and sequential consistency are incomparable

  => *quiescent consistency does not necessarily preserve program order*
  => *sequential consistency is unaffected by quiescent periods*

- a correctness condition C is compositional if whenever every object satisfies C, the system as a whole satisfies C

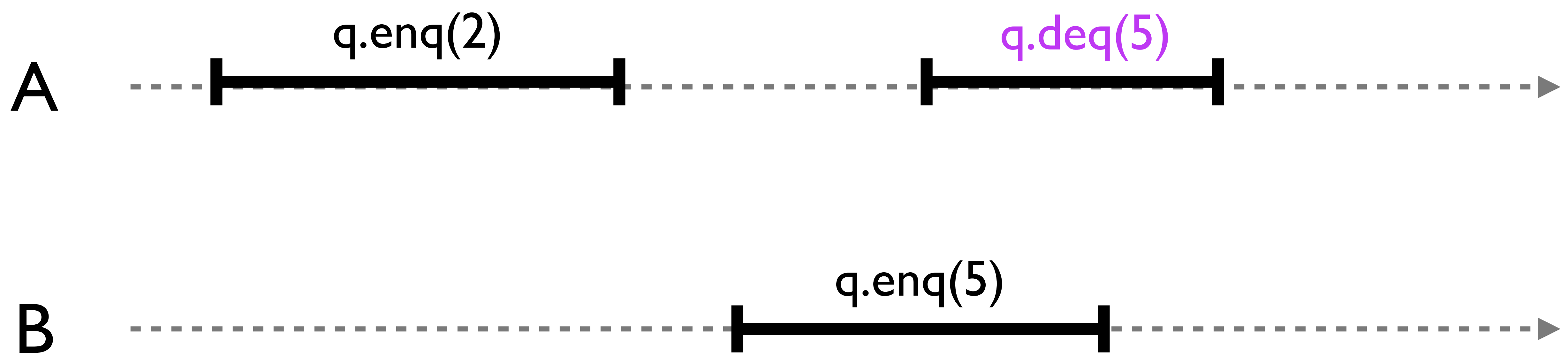
  => *quiescent consistency is compositional*
  => *sequential consistency, unfortunately, is not compositional*

Next on the agenda

1. concurrent objects and correctness
2. quiescent and sequential consistency
3. linearizability

Strengthening sequential consistency to gain compositionality

- should this (sequentially consistent) behaviour be allowed?



Linearizability

each method call should appear to take effect instantaneously at some moment <u>between its invocation and response</u>

an <u>object</u> is linearizable if all of its possible executions are linearizable

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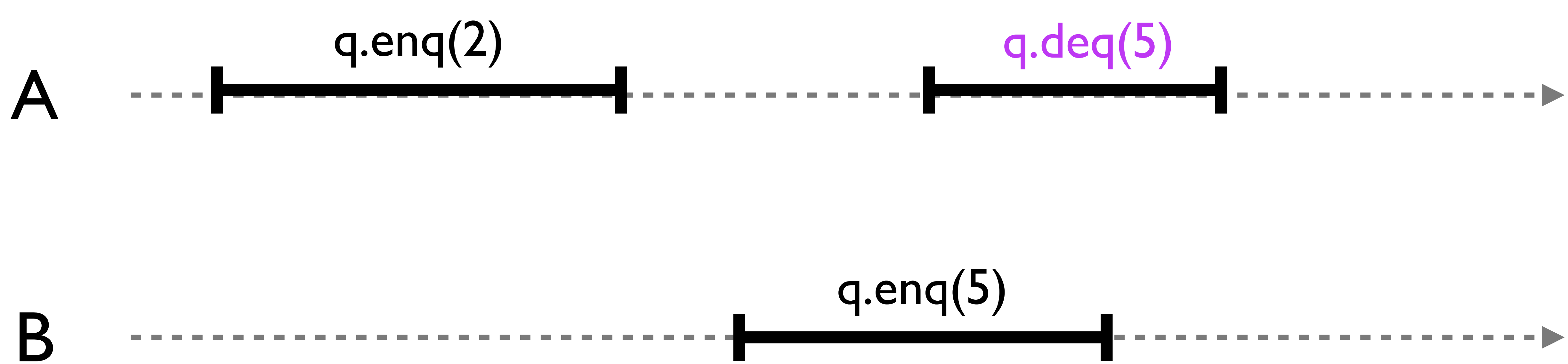

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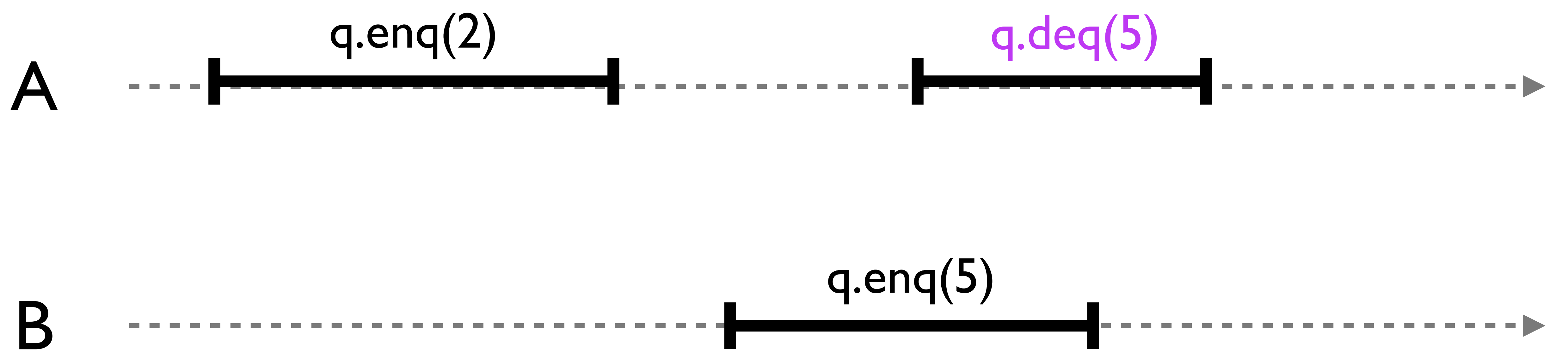

X sequentially consistent, but not linearizable

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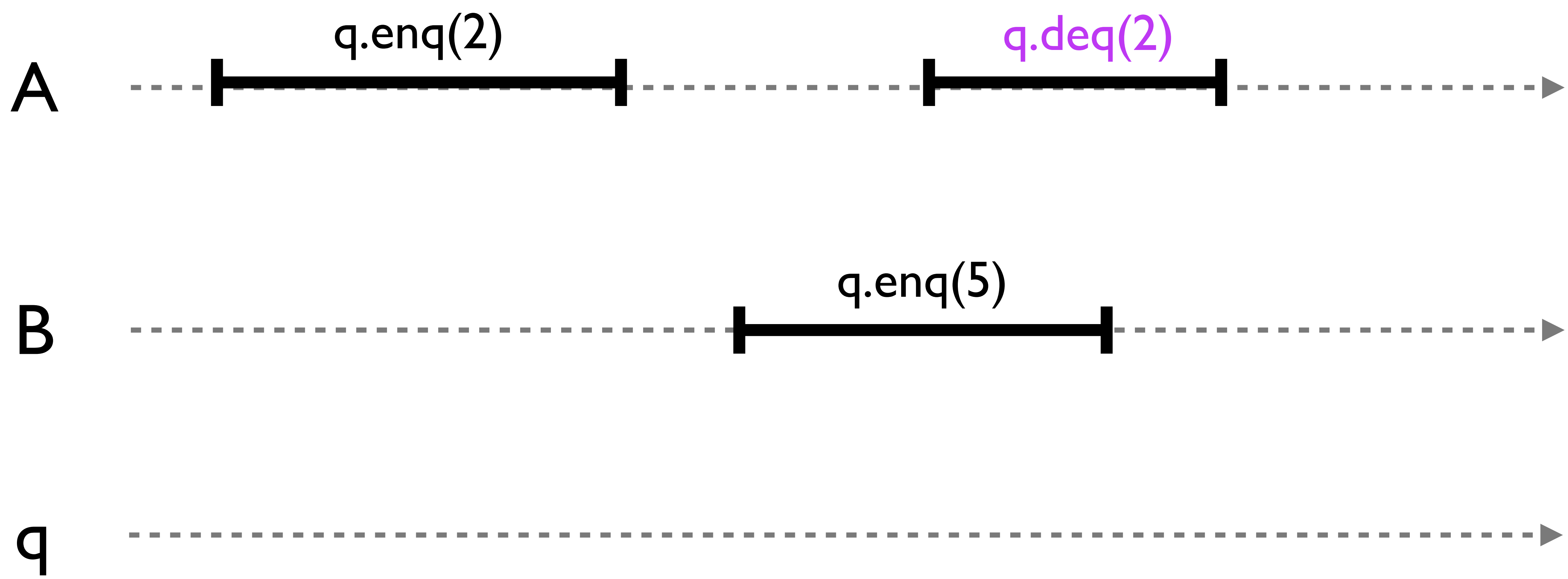

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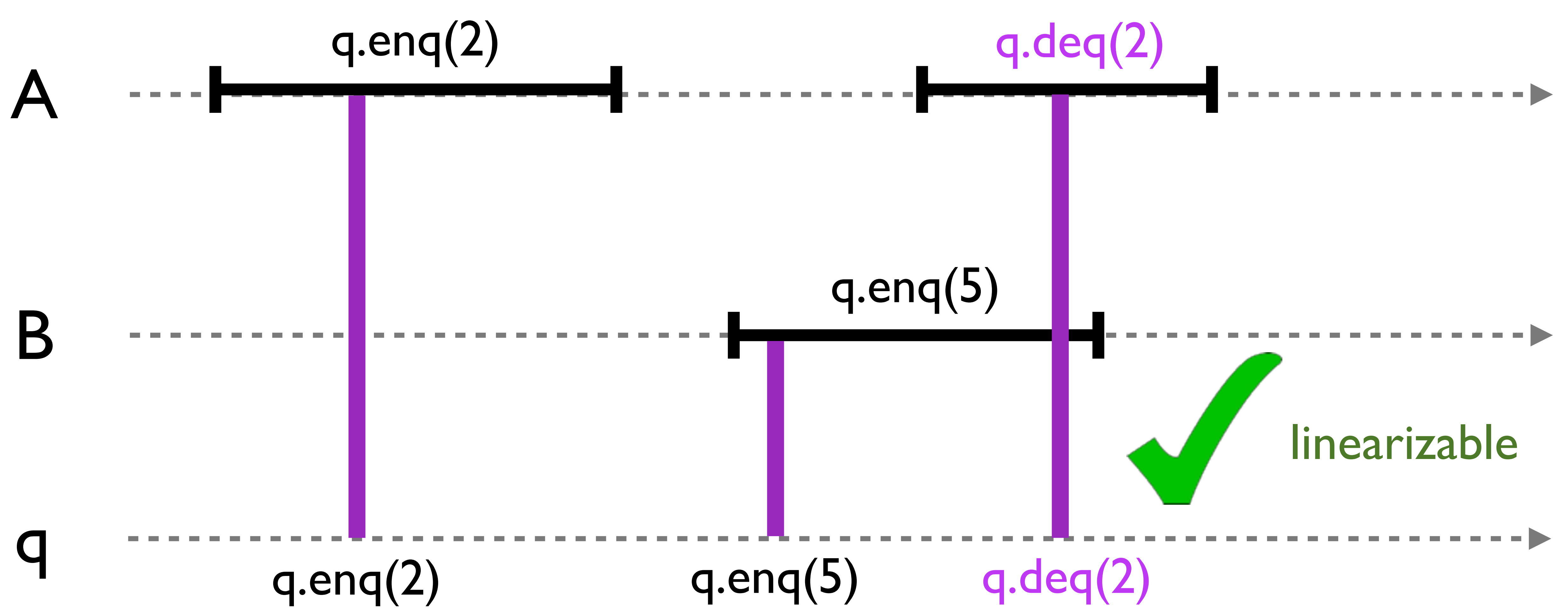
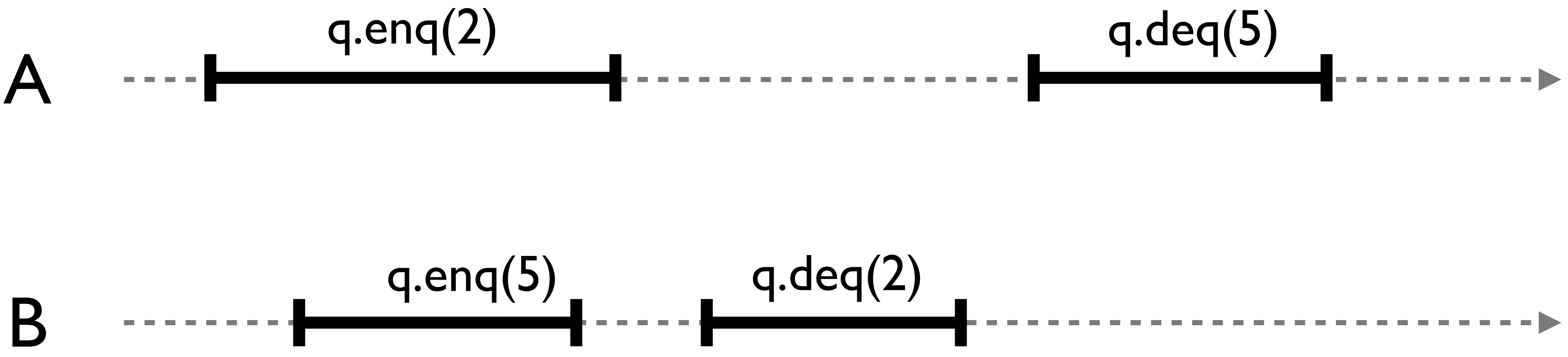


Linearizability: examples



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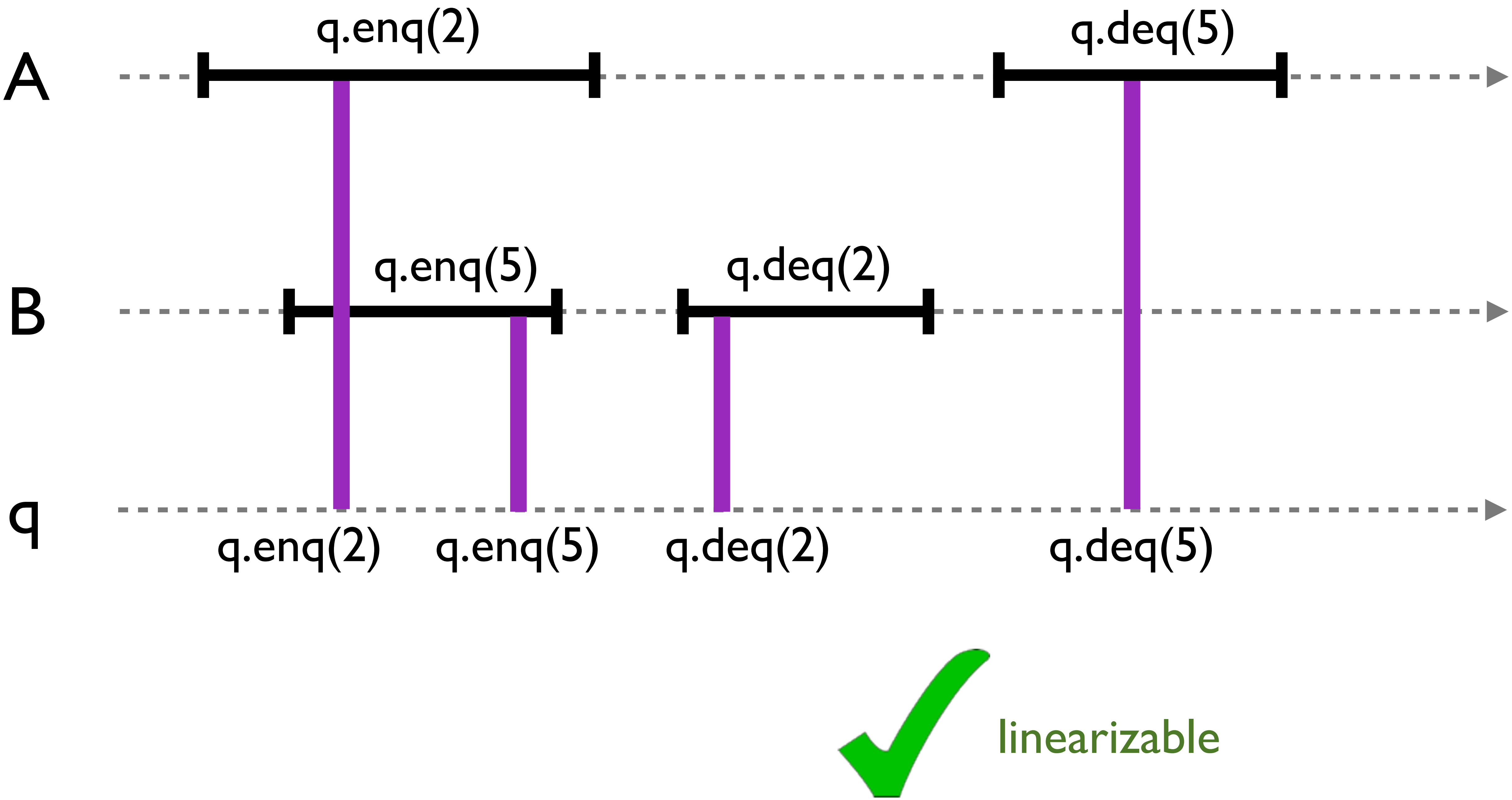

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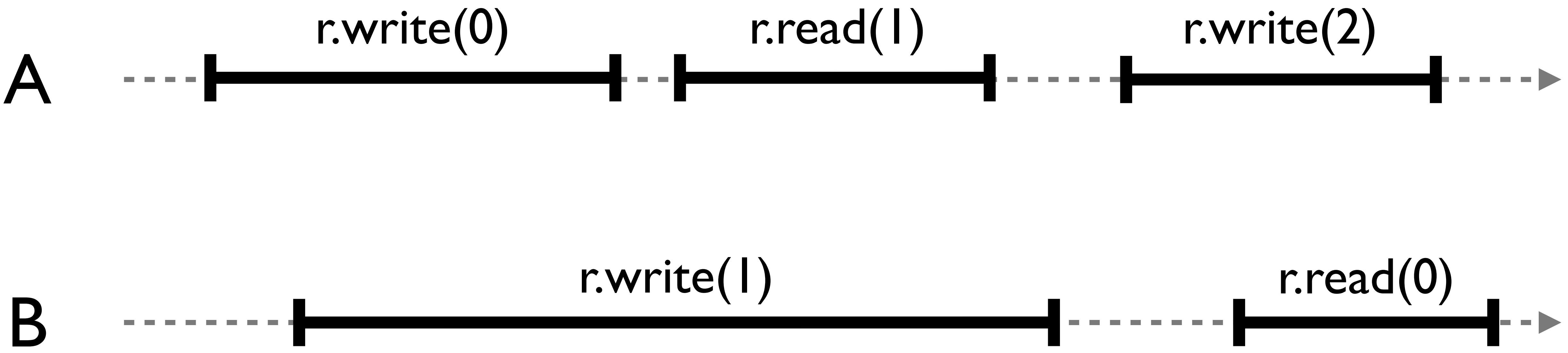

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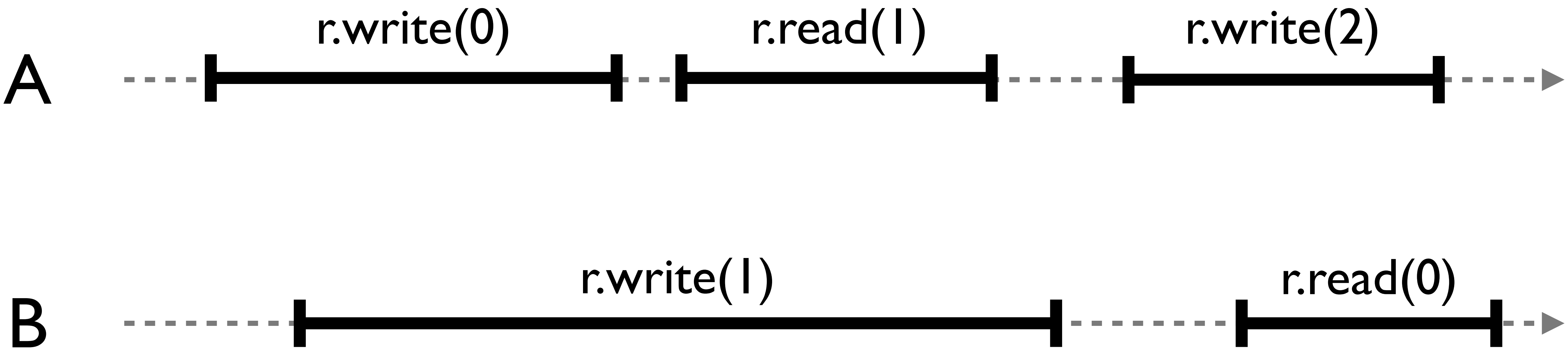
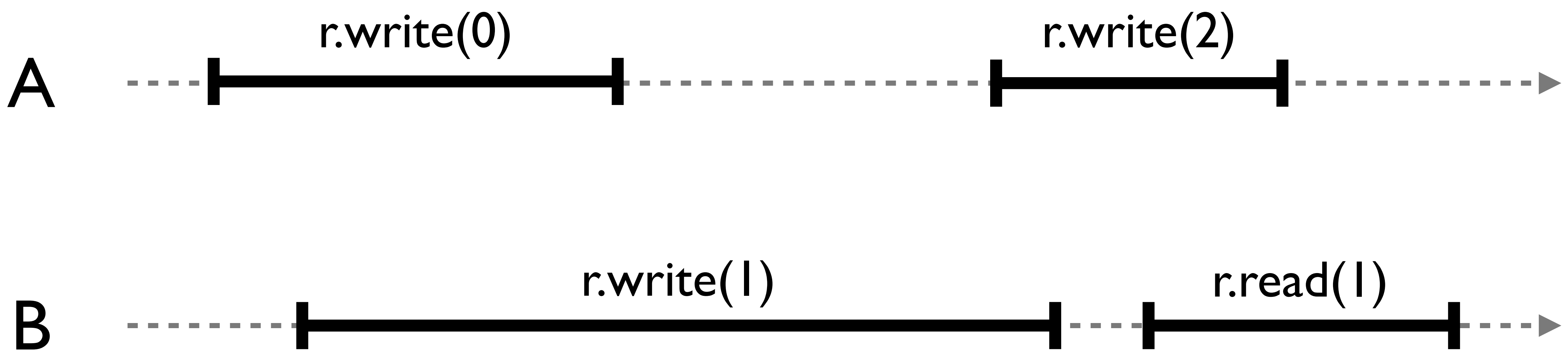


X not sequentially consistent, not linearizable

Linearizability: examples



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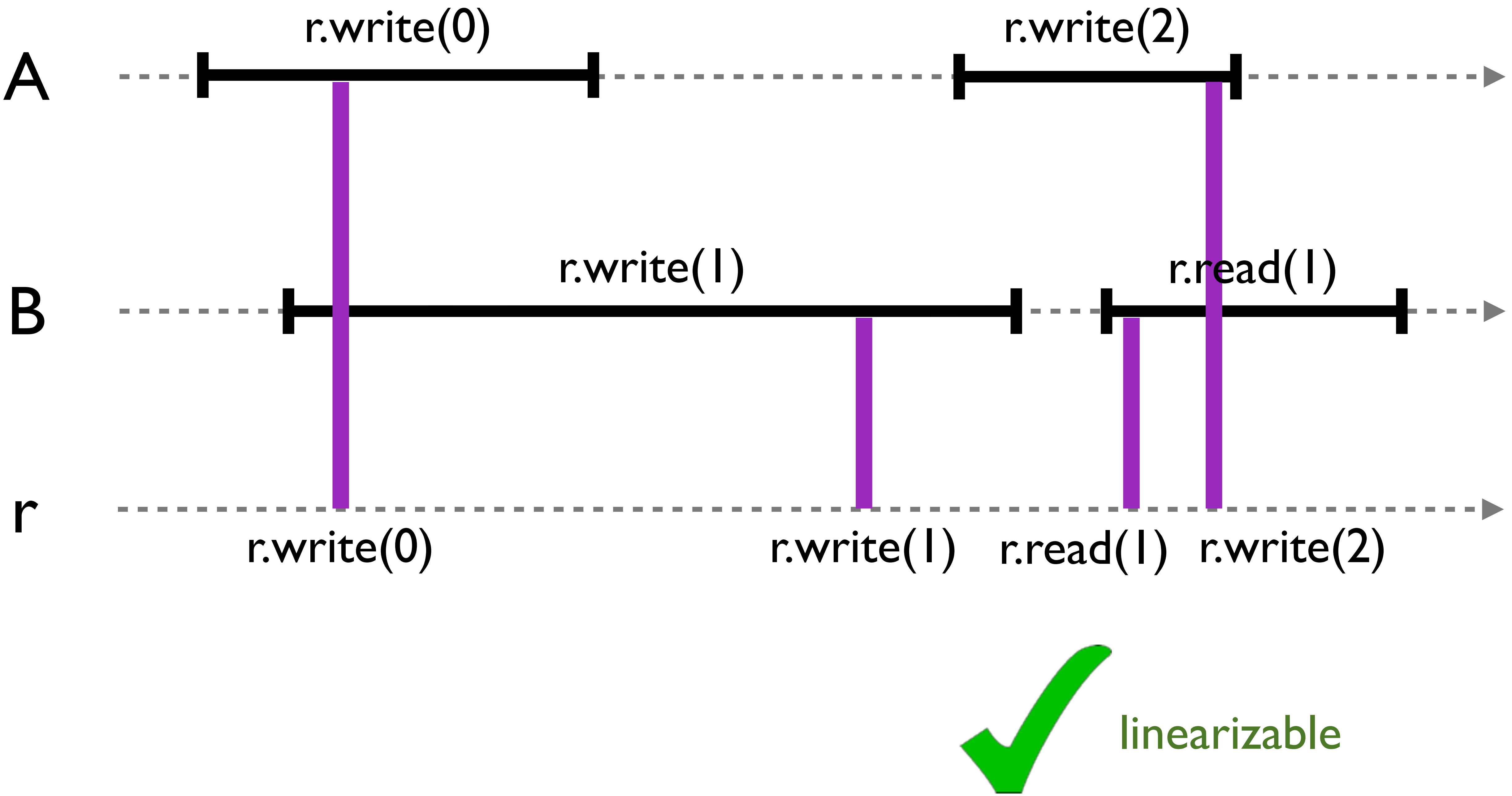

Linearization points

- to show that a concurrent object is linearizable, one must identify for each method a linearization point where the method takes effect
- for lock-based objects, these are the critical sections
- for lock-free approaches, the linearization point is a single step where they effects of the method call become visible to other method calls

Sequential consistency vs. linearizability

- linearizable executions are also sequentially consistent
- sequential consistency is less restrictive: allows method calls to take effect after their response
- linearizability is compositional: the result of composing linearizable objects is linearizable

Formal definitions

- a call of an operation is split into two events:

  invocation [A q.op(a_1, ..., a_n)]
  response [A q:Ok(r)]

- where A is a thread ID, q an object, op(a_1, ..., a_n) an invocation of call with arguments, and Ok(r) a successful response of call with result r

- a history is a sequence of invocation / response events

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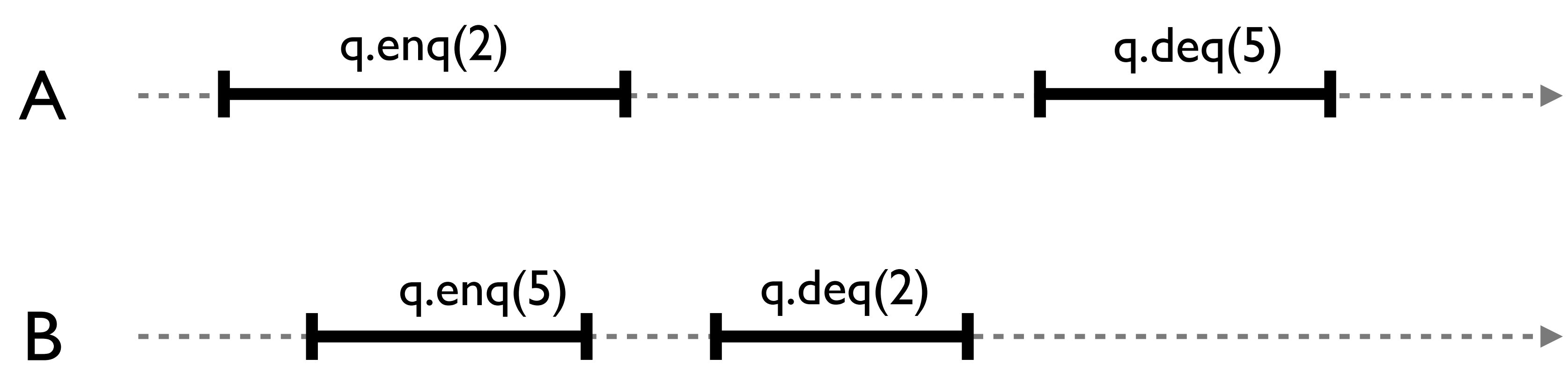
H = [A q.enq(2)], [B q.enq(5)], [B q.Ok], [A q.Ok], [B q.deq()], [B q.Ok(2)], [A q.deq()], [A q.Ok(5)]



Formal definitions

- we can define projections on objects and on threads

- assume we have a history:

H = [A q1.enq(2)], [B q2.enq(5)], [B q2.Ok], [A q1.Ok],
[B q1.deq()], [B q1.Ok(2)], [A q2.deq()], [A q2.Ok(5)]

- object projection:

H|q1 = [A q1.enq(2)], [A q1.Ok], [B q1.deq()], [B q1.Ok(2)]

- thread projection:

H|A = [A q1.enq(2)], [A q1.Ok], [A q2.deq()], [A q2.Ok(5)]

Formal definitions

- a response matches an invocation if their object and thread names agree

- a history is sequential if it starts with an invocation, and each invocation (except possibly the last) is immediately followed by a matching response

H = [A q.enq(2)], [A q.Ok], [B q.enq(5)], [B q.Ok]

- a sequential history is legal if it agrees with the sequential specification of each object

Formal definitions

- a call op_1 precedes another call op_2 ($op_1 \rightarrow op_2$) if op_1's response event occurs before op_2's invocation event

- we write $\rightarrow_H$ for the precedence relation induced by H

  => *e.g.* $q.enq(2) \rightarrow_H q.enq(5)$

- an invocation is pending if it has no matching response

- a history is complete if it does not have pending responses

- complete(H) is the subhistory of H with all pending invocations removed

Linearizability: the definition

- two histories H and G are equivalent if H|A = G|A for all threads A

- a history H is linearizable if it can be extended to a history G by adding zero or more response events, such that:

  => *complete(G) is equivalent to some legal sequential history S*

  => $\rightarrow_H \subseteq \rightarrow_S$ *(i.e. the precedences of H are maintained)*

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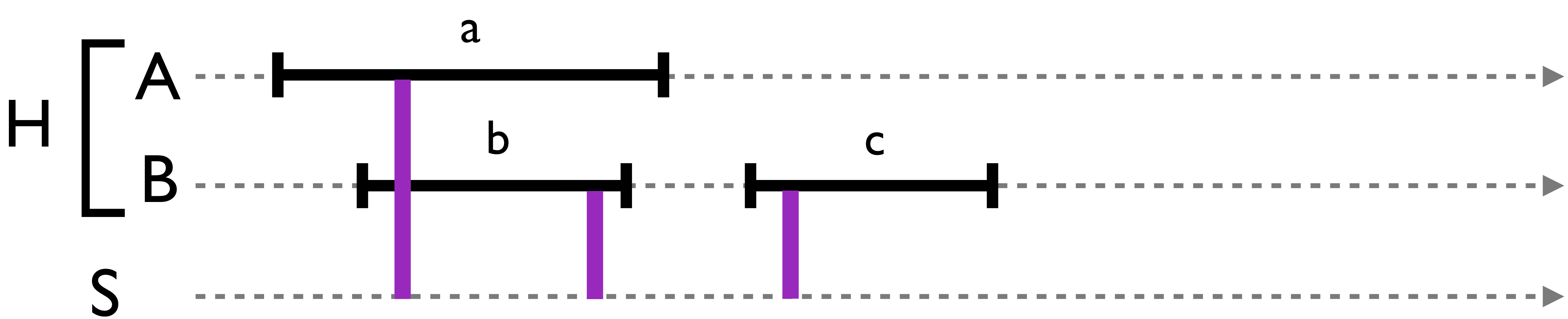


$\rightarrow_H = \{a \rightarrow c, b \rightarrow c\}$ $\rightarrow_S = \{a \rightarrow b, a \rightarrow c, b \rightarrow c\}$

Next on the agenda

1. concurrent objects and correctness ✓
2. quiescent and sequential consistency ✓
3. linearizability ✓

Final remarks

- correctness notions for concurrent objects boil down to "equivalences" with sequential computations

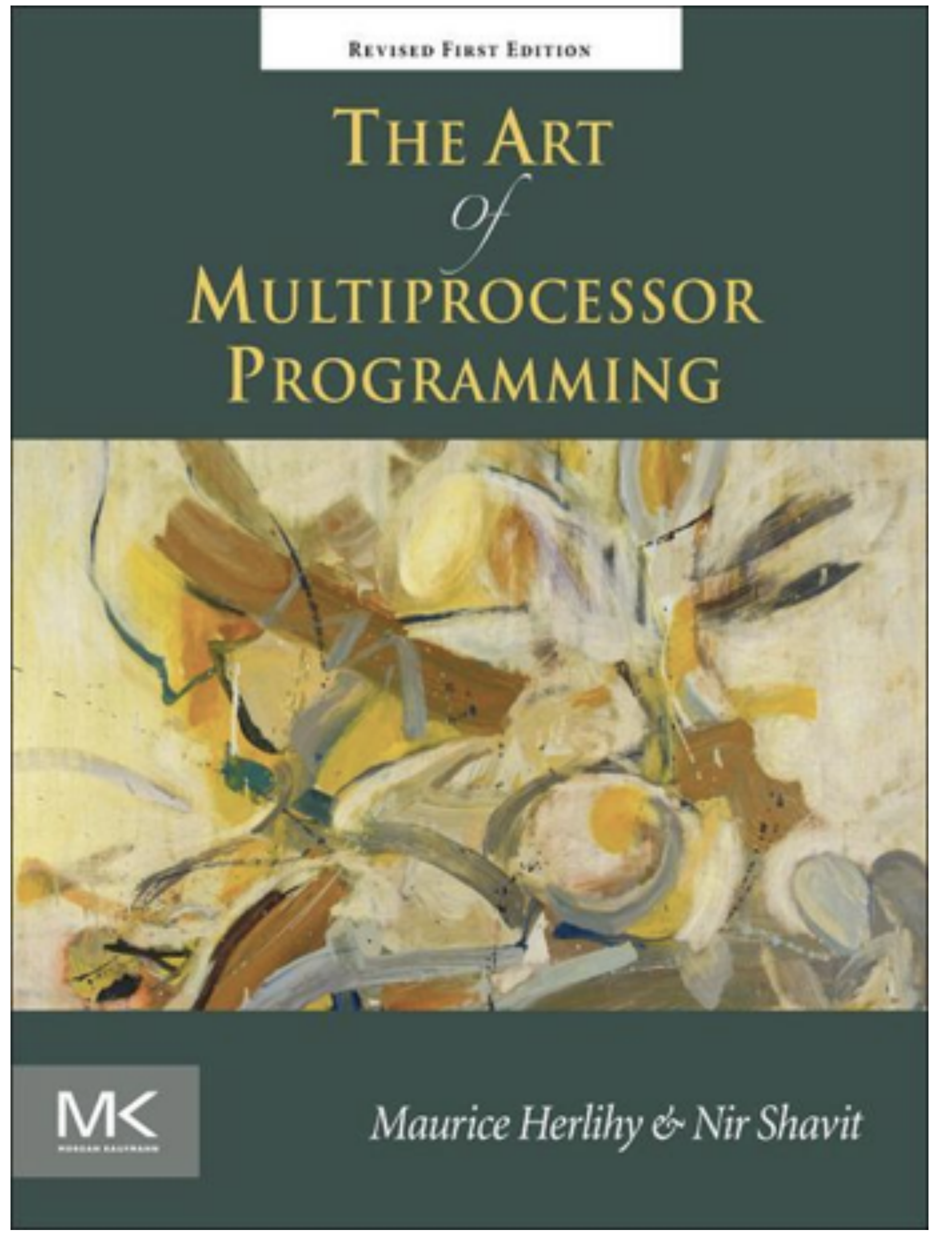
  => *quiescent consistency, sequential consistency, linearizability*
  => *objects we built in previous weeks were linearizable*

- correctness conditions depend on the application's needs

- in most modern multiprocessor architectures, memory reads/writes are not sequentially consistent

  => *too expensive!*
  => *must "ask" for it explicitly when needed*

Lecture based on Chapter 3 of:



recommended reading!