Last week: synchronisation, but lacking the simplicity

• we looked at various solutions to the mutual exclusion problem

• algorithms were limited to the simplest tools - atomic read and write to shared memory

  => difficult to implement; complex
  => reliance on busy waiting
  => no encapsulation of synchronisation variables
Short diversion: hot desks
Short diversion: hot desks

desk please!
Short diversion: hot desks

merci!
Short diversion: hot desks

merci!
Short diversion: hot desks
Short diversion: hot desks

1 desk please!
Short diversion: hot desks

super duper!
Short diversion: hot desks

super duper!
Short diversion: hot desks
Short diversion: hot desks

[Image of stick figures and desks with one saying "gotta wait!"

0]
Short diversion: hot desks

*all done!*

*gotta wait!*

0
Short diversion: hot desks

gotta wait!

all done!
Short diversion: hot desks

gotta wait!

all done!
Short diversion: hot desks

[gotta wait!]

1
Short diversion: hot desks

woohoo!
Short diversion: hot desks

woohoo!
Short diversion: hot desks

0

a semaphore
Today’s lecture: semaphores

• we will discuss semaphores, an important synchronisation primitive

• conceptually simple, although their implementations require stronger atomic operations

• widespread use in operating systems

• invented by Dijkstra in 1965
Next on the agenda

1. general and binary semaphores

2. implementing semaphores

3. beyond the mutual exclusion problem

4. simulating general semaphores
General semaphores
(aka “counting semaphores”)

• a general semaphore is an object consisting of:

  (1) an integer variable $\textit{count}$ such that $\textit{count} \geq 0$

  (2) two atomic operations: $\textit{down}$ and $\textit{up}$

if a process calls $\textit{down}$ when $\textit{count} > 0$, then $\textit{count}$ is decremented by 1 (otherwise it first waits)

if a process calls $\textit{up}$, then $\textit{count}$ is incremented by 1
General semaphores
(in Eiffel-like pseudocode)

class SEMAPHORE

feature
  count : INTEGER

  down
do-atOMIC
    await count > 0
    count := count - 1
  end

  up
do-atOMIC
    count := count + 1
  end
end
class SEMAPHORE

feature
  count : INTEGER

  down
    do-atomic
    await count > 0
    count := count - 1
  end

  up
    do-atomic
    count := count + 1
  end

end

will discuss how to implement atomicity of “test and decrement”, and how to avoid busy wait later!
Mutual exclusion for two processes

• create a semaphore $s$ and initialise $s.count$ to 1; then:

\[
\text{s.down} \\
\text{critical section} \\
\text{s.up}
\]
Mutual exclusion for two processes

- create a semaphore $s$ and initialise $s.count$ to 1; then:

  - $s.down$  
  - critical section  
  - $s.up$

  one process at a time; or one hot desk!
Mutual exclusion for two processes

- or in the style of last week’s mutual exclusion problems:

<table>
<thead>
<tr>
<th>count := 1</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>P1</th>
<th>P2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>while true loop</td>
</tr>
<tr>
<td></td>
<td>await count &gt; 0</td>
</tr>
<tr>
<td></td>
<td>count := count - 1</td>
</tr>
<tr>
<td>2</td>
<td>critical section</td>
</tr>
<tr>
<td>3</td>
<td>count := count + 1</td>
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<tr>
<td>4</td>
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<td></td>
<td>end</td>
</tr>
<tr>
<td></td>
<td>end</td>
</tr>
</tbody>
</table>
Mutual exclusion for two processes

• mutual exclusion and deadlock freedom can be proven
  => remember the atomicity of down and up!

• solution does not satisfy starvation freedom
  => a different implementation later will fix this
The general semaphore invariant

• general semaphores are characterised by the following 
  invariant -- important for proofs!

• given some semaphore, let:

  => $k$ denote its initial value with $k \geq 0$
  => $\text{count}$ denote its current value
  => $\#\text{down}$ denote the number of completed down operations
  => $\#\text{up}$ denote the number of completed up operations

• then the following equations are invariant:

  (1) $\text{count} \geq 0$
  (2) $\text{count} = k + \#\text{up} - \#\text{down}$
Binary semaphores

• in the previous example, \( s.count \) is always either 0 or 1

• such a semaphore is called a binary semaphore and can be implemented using a Boolean variable

\[
\begin{align*}
b & : \text{BOOLEAN} \\
\text{down} & \\
& \text{do-atomic} \\
& \text{await} \ b \\
& \quad b := \text{false} \\
& \text{end} \\
\text{up} & \\
& \text{do-atomic} \\
& \quad b := \text{true} \\
& \text{end}
\end{align*}
\]
Binary semaphores

• in the previous example, \( s.count \) is always either 0 or 1

• such a semaphore is called a binary semaphore and can be implemented using a Boolean variable

\[
\begin{align*}
b : \text{BOOLEAN} \\
\text{down} \\
\text{do-atomic} \\
\text{await } b \\
b := \text{false} \\
\text{end} \\
\text{up} \\
\text{do-atomic} \\
b := \text{true} \\
\text{end}
\end{align*}
\]

This is deceptively similar to the previous week’s early, and wrong attempts at providing mutual exclusion. What’s different?
Next on the agenda

1. general and binary semaphores

2. implementing semaphores

3. beyond the mutual exclusion problem

4. simulating general semaphores
Avoiding busy waiting

• **busy-wait semaphores** are not ideal

  => *they are not starvation free*
  => *inefficient in the context of multitasking*

• more preferable would be for processes to **block themselves** when having to wait

  => *thus freeing processing resources as early as possible*

• idea: keep track of blocked processes, “waking them” upon **up** calls on the semaphore
Efficiency: blocking of processes

• A process can be in the following states:
  • new: being created.
  • running: instructions are being executed.
  • blocked: currently waiting for an event.
  • ready: ready to be executed, but not been assigned a processor yet.
  • terminated: finished executing.

Avoiding busy waiting
Avoiding busy waiting

A process can be in the following states:

- **new**: being created.
- **running**: instructions are being executed.
- **blocked**: currently waiting for an event.
- **ready**: ready to be executed, but not been assigned a processor yet.
- **terminated**: finished executing.

Context switch

Avoiding busy waiting

if s.count < 1
Implementing the scheme

• to avoid starvation, we will track blocked processes in a collection \textit{blocked}

• we equip \textit{blocked} with the following operations, which will be integrated into \textit{down} and \textit{up}

  => \textit{add(P)} \quad -- \textit{insert process P into collection}
  => \textit{remove} \quad -- \textit{select, remove, and return an item from the collection}
  => \textit{is\_empty} \quad -- \textit{true if collection empty; false otherwise}

• if \textit{blocked} is implemented as a \textit{set}, we call the semaphore \textit{weak}; if as a \textit{FIFO queue}, then \textit{strong}
Weak semaphore

- a weak semaphore is a blocking semaphore in which the collection `blocked` is implemented as a set

=> blocked.remove will pick and remove a random process from blocked

\[
\text{down} \\
\text{do-atomic} \\
\quad \text{if count} > 0 \text{ then} \\
\quad \quad \text{count} := \text{count} - 1 \\
\quad \text{else} \\
\quad \quad \text{blocked.add}(P) \\
\quad \quad P.\text{state} := \text{blocked} \\
\quad \text{end} \\
\text{end} \\
\]

\[
\text{up} \\
\text{do-atomic} \\
\quad \text{if blocked.is_empty} \text{ then} \\
\quad \quad \text{count} := \text{count} + 1 \\
\quad \text{else} \\
\quad \quad Q := \text{blocked.remove} \\
\quad \quad Q.\text{state} := \text{ready} \\
\quad \text{end} \\
\text{end} \\
\]
Weak semaphore

- a **weak semaphore** is a blocking semaphore in which the collection `blocked` is implemented as a **set**

=> `blocked.remove` will *pick and remove a random process from blocked*

```plaintext
down
  do-atomic
    if count > 0 then
      count := count - 1
    else
      blocked.add(P)
      P.state := blocked
    end
  end
end

up
  do-atomic
    if blocked.is_empty then
      count := count + 1
    else
      Q := blocked.remove
      Q.state := ready
    end
  end

-- add current process P to blocked
-- block P (instead of busy wait)
```
• a **weak semaphore** is a blocking semaphore in which the collection `blocked` is implemented as a **set**

=> `blocked.remove` will **pick and remove a random process from blocked**

– *select and remove some process Q from blocked*
– *unblock Q so that it can access the resource*  
(Question: why is `count` left unchanged?)
Mutual exclusion for two processes

• weak semaphores provide starvation-freedom in the two process scenario

=> why?

• what about mutual exclusion for $n$ processes?
Mutual exclusion for \( n \) processes

- create a semaphore \( s \) and initialise \( s\).\text{count} \text{ to } 1; \text{ then:}

\[
\begin{align*}
&\text{s.down} \\
&\text{critical section} \\
&\text{s.up}
\end{align*}
\]

- starvation is possible for \( n > 2 \) with weak semaphores because we select a process from \( \text{blocked} \) at random

- solution is to use a strong semaphore, in which \( \text{blocked} \) is implemented as a FIFO queue
Strong semaphores provide a solution to the mutual exclusion problem with \( n \) processes (how to prove)

- **mutual exclusion** -- prove that the following is invariant:
  \[
  \#cs + \text{count} = 1
  \]
  where \( \#cs \) is the number of processes in critical sections

- **starvation freedom** -- apply *proof by contradiction*
  
  \( \Rightarrow \) begin by assuming that a process in blocked is starved

- see *Theorem 4.6* in the course notes
A note on implementing atomicity

• you will typically never have to implement the atomic down and up operations of a semaphore yourself

  => provided, e.g. in Java

• down and up can be built in software from lower-level primitives, using e.g. synchronisation algorithms

• alternatively:

  => using “test-and-set” instructions
      (atomic read and write – see later lecture)
  => disabling interrupts (only realistic on a single processing unit)
A note on semaphores in Java

• java.util.concurrent.Semaphore

  http://docs.oracle.com/javase/8/docs/api/java/util/concurrent/Semaphore.html

• constructors

  => Semaphore(int k)  -- a weak semaphore
  => Semaphore(int k, boolean b)  -- a strong semaphore if b true

• operations

  => acquire()  -- corresponds to down
  => release()  -- corresponds to up
Next on the agenda

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The $k$-exclusion problem

- in the $k$-exclusion problem, we allow up to $k$ processes to simultaneously be in their critical sections

  => mutual exclusion is the $k = 1$ instance

- use a general semaphore corresponding to the number of processes allowed to be in their critical sections

```
  s.count := k

  P_i

  while true loop
    s.down
    critical section
    s.up
    non-critical section
  end
```
The \( k \)-exclusion problem

- in the \( k \)-exclusion problem, we allow up to \( k \) processes to simultaneously be in their critical sections
  
  \( \Rightarrow \) mutual exclusion is the \( k = 1 \) instance

- use a general semaphore corresponding to the number of processes allowed to be in their critical sections

\[
\begin{array}{|c|c|}
\hline
\text{s.count} & := \text{k} \\
\hline
\text{P}_i \\
\hline
\text{while true loop} & \\
1 & \text{s.down} \\
2 & \text{critical section} \\
3 & \text{s.up} \\
4 & \text{non-critical section} \\
\text{end} \\
\hline
\end{array}
\]
Barriers

• semaphores can be used to control the ordering of events in a system

• a barrier is a form of synchronisation that determines a point in a program’s execution that all processes in a group have to reach before any of them may move on

=> important for concurrent iterative algorithms
## Barriers

- Semaphores can be used to control the **ordering of events** in a system.

- A **barrier** is a form of synchronisation that determines a point in a program’s execution that all processes in a group have to reach before any of them may move on.

  => *important for concurrent iterative algorithms*

<table>
<thead>
<tr>
<th></th>
<th>P1</th>
<th>P2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>code before the barrier</td>
<td>code before the barrier</td>
</tr>
<tr>
<td>2</td>
<td>s1.up</td>
<td>s2.up</td>
</tr>
<tr>
<td>3</td>
<td>s2.down</td>
<td>s1.down</td>
</tr>
<tr>
<td>4</td>
<td>code after the barrier</td>
<td>code after the barrier</td>
</tr>
</tbody>
</table>
• semaphores can be used to control the ordering of events in a system

• a barrier is a form of synchronisation that determines a point in a program’s execution that all processes in a group have to reach before any of them may move on

=> important for concurrent iterative algorithms

---

**s1 is the barrier for P2; s2 is the barrier for P1 -- why are they initialised to 0?**

<table>
<thead>
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<tbody>
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</tr>
<tr>
<td>3 s2.down</td>
<td>3 s1.down</td>
</tr>
<tr>
<td>4 code after the barrier</td>
<td>4 code after the barrier</td>
</tr>
</tbody>
</table>
The producer-consumer problem

Producers

store (buffer, int)

Buffer

243
46
71
97

consume (buffer)

Consumers
The producer-consumer problem

Producers

store (buffer, int)

Buffer

243
46
71
97

Consumers

consume (buffer)

require buffer.not_full

require buffer.not_empty
The producer-consumer problem

• a good solution would:
  
  $\Rightarrow$ ensure that every data item produced is eventually consumed
  $\Rightarrow$ be deadlock-free
  $\Rightarrow$ be starvation-free

• need a semaphore for mutual exclusion (the buffer)

• but additional semaphore(s) for condition synchronisation
  
  $\Rightarrow$ e.g. consumer should block until the buffer is non-empty
Solution for an unbounded buffer

\[
\begin{align*}
\text{mutex.count} & := 1 \\
\text{not_empty.count} & := 0 \\
\end{align*}
\]

\[
\begin{array}{|c|c|}
\hline
\text{Producer}_i & \text{Consumer}_i \\
\hline
1 & \text{while true loop} \\
2 & \quad d := \text{produce} \\
3 & \quad \text{mutex.down} \\
4 & \quad b.\text{append}(d) \\
5 & \quad \text{mutex.up} \\
6 & \quad \text{not_empty.up} \\
7 & \quad \text{end} \\
8 & \text{while true loop} \\
9 & \quad \text{not_empty.down} \\
10 & \quad \text{mutex.down} \\
11 & \quad d := b.\text{remove} \\
12 & \quad \text{mutex.up} \\
13 & \quad \text{consume}(d) \\
14 & \quad \text{end} \\
\hline
\end{array}
\]
## Solution for an _unbounded_ buffer

```plaintext
mutex.count := 1
not_empty.count := 0
```

<table>
<thead>
<tr>
<th>Producer(_i)</th>
<th>Consumer(_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
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<td>3</td>
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<td>5</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>while true loop</th>
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</tr>
</thead>
<tbody>
<tr>
<td>d := produce</td>
<td>not_empty.down</td>
</tr>
<tr>
<td>mutex.down</td>
<td>mutex.down</td>
</tr>
<tr>
<td>b.append(d)</td>
<td>d := b.remove</td>
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<tr>
<td>mutex.up</td>
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<td>not_empty.up</td>
<td>consume(d)</td>
</tr>
<tr>
<td>end</td>
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</tr>
</tbody>
</table>

*observe that not_empty.count = \#items_in_buffer*
**Solution for an unbounded buffer**

Observe that `not_empty.count = #items_in_buffer`

<table>
<thead>
<tr>
<th>Producer$_i$</th>
<th>Consumer$_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="chart.png" alt="Diagram showing the solution process" /></td>
<td><img src="chart.png" alt="Diagram showing the solution process" /></td>
</tr>
</tbody>
</table>

- `mutex.count := 1`
- `not_empty.count := 0`

1. **Producer**
   - `while true loop`
   - `d := produce`
   - `mutex.down`
   - `b.append(d)`
   - `mutex.up`
   - `not_empty.up`

2. **Consumer**
   - `while true loop`
   - `not_empty.down`
   - `mutex.down`
   - `d := b.remove`
   - `mutex.up`
   - `consume(d)`
   - `not_empty.up`

blocks until `not_empty.count > 0`
Solution for a bounded buffer

```plaintext
mutex.count := 1
not_empty.count := 0
not_full.count := k
```

<table>
<thead>
<tr>
<th>Producer \textsubscript{i}</th>
<th>Consumer \textsubscript{i}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>while true loop</td>
</tr>
<tr>
<td>2</td>
<td>d := produce</td>
</tr>
<tr>
<td>3</td>
<td>not_full.down</td>
</tr>
<tr>
<td>4</td>
<td>mutex.down</td>
</tr>
<tr>
<td>5</td>
<td>b.append(d)</td>
</tr>
<tr>
<td></td>
<td>mutex.up</td>
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<tr>
<td></td>
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<tr>
<td></td>
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Solution for a bounded buffer

mutex.count := 1
not_empty.count := 0
not_full.count := k

Producer$_i$

1
while true loop
  d := produce
  not_full.down
  mutex.down
  b.append(d)
  mutex.up
  not_empty.up
end

Consumer$_i$

1
while true loop
  not_empty.down
  mutex.down
  d := b.remove
  mutex.up
  not_full.up
  consume(d)
end

where $k$ is the size of the buffer
Dining philosophers problem
(a solution that can deadlock)

- multiple semaphores must be used with care -- they are prone to deadlock!

```
s[1].count := 1, ..., s[n].count := 1

<table>
<thead>
<tr>
<th>Philosopher_i</th>
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<tbody>
<tr>
<td>while true loop</td>
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<td>6</td>
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end
```
Dining philosophers problem
(a solution that can deadlock)

- multiple semaphores must be used with care -- they are prone to deadlock!

\[
s[1].\text{count} := 1, \ldots, s[n].\text{count} := 1
\]

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<td>\par 6</td>
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\textbf{circular waiting!}
Dining philosophers problem
\((an \text{ asymmetric fix!})\)

- assume that philosopher \(n\) picks up the left fork before the right fork

- this breaks the circle of resource requests; there will always be one philosopher who can acquire both forks and release them again

<table>
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<td><strong>while true loop</strong></td>
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Next on the agenda

1. general and binary semaphores
2. implementing semaphores
3. beyond the mutual exclusion problem
4. simulating general semaphores
General semaphores are superfluous

- while conceptually useful, general semaphores (theoretically) are not necessary -- they can be implemented through binary semaphores alone
General semaphores are superfluous

mutex.count := 1 -- binary semaphore
delay.count := 1 -- binary semaphore
count := k

general_down
do
    delay.down
    mutex.down
    count := count - 1
    if count > 0 then
        delay.up
    end
    mutex.up
end

general_up
do
    mutex.down
    count := count + 1
    if count = 1 then
        delay.up
    end
    mutex.up
end
General semaphores are superfluous

mutex.count := 1  -- binary semaphore
delay.count := 1   -- binary semaphore
count := k

value of the general semaphore

General down:

do
  delay.down
  mutex.down
  count := count - 1
  if count > 0 then
    delay.up
  end
  mutex.up
end

General up:

do
  mutex.down
  count := count + 1
  if count = 1 then
    delay.up
  end
  mutex.up
end

protects count

not called when count = 0
Next on the agenda

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Summary

- semaphores are **conceptually simple** but powerful tools for solving synchronisation problems

- choice of implementation can affect starvation-freedom

- applications **beyond mutual exclusion**: $k$-exclusion, barriers, condition synchronisation

**but**: correct usage is still **far from trivial**

- essential reading: *Chapter 4 of the CCC textbook*